Mplus Short Courses
Topic 6

Categorical Latent Variable Modeling Using Mplus: Longitudinal Data

Linda K. Muthén
Bengt Muthén

www.statmodel.com
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Mplus Background

- Inefficient dissemination of statistical methods:
  - Many good methods contributions from biostatistics, psychometrics, etc are underutilized in practice
- Fragmented presentation of methods:
  - Technical descriptions in many different journals
  - Many different pieces of limited software
- Mplus: Integration of methods in one framework
  - Easy to use: Simple, non-technical language, graphics
  - Powerful: General modeling capabilities
- Mplus versions
  - V1: November 1998
  - V2: February 2001
  - V3: March 2004
  - V4: February 2006
  - V5: November 2007
  - V5.2: November 2008
- Mplus team: Linda & Bengt Muthén, Thuy Nguyen, Tihomir Asparouhov, Michelle Conn

Statistical Analysis With Latent Variables
A General Modeling Framework

Statistical Concepts Captured By Latent Variables

Continuous Latent Variables

- Measurement errors
- Factors
- Random effects
- Frailties, liabilities
- Variance components
- Missing data

Categorical Latent Variables

- Latent classes
- Clusters
- Finite mixtures
- Missing data

Statistical Analysis With Latent Variables
A General Modeling Framework (Continued)

Models That Use Latent Variables

Continuous Latent Variables

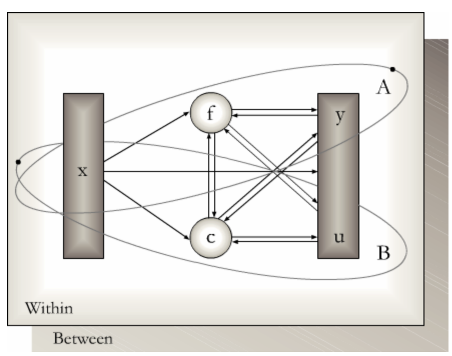
- Factor analysis models
- Structural equation models
- Growth curve models
- Multilevel models

Categorical Latent Variables

- Latent class models
- Mixture models
- Discrete-time survival models
- Missing data models

Mplus integrates the statistical concepts captured by latent variables into a general modeling framework that includes not only all of the models listed above but also combinations and extensions of these models.

General Latent Variable Modeling Framework



- Observed variables
  - x background variables (no model structure)
  - y continuous and censored outcome variables
  - u categorical (dichotomous, ordinal, nominal) and count outcome variables
- Latent variables
  - f continuous variables
    - – interactions among f's
  - c categorical variables
    - – multiple c's

Mplus

Several programs in one

- Exploratory factor analysis
- Structural equation modeling
- Item response theory analysis
- Latent class analysis
- Latent transition analysis
- Survival analysis
- Growth modeling
- Multilevel analysis
- Complex survey data analysis
- Monte Carlo simulation

Fully integrated in the general latent variable framework

Overview Of Mplus Courses

- **Topic 1.** August 20, 2009, Johns Hopkins University: Introductory - advanced factor analysis and structural equation modeling with continuous outcomes
- **Topic 2.** August 21, 2009, Johns Hopkins University: Introductory - advanced regression analysis, IRT, factor analysis and structural equation modeling with categorical, censored, and count outcomes
- **Topic 3.** March, 2010, Johns Hopkins University: Introductory and intermediate growth modeling
- **Topic 4.** March, 2010, Johns Hopkins University: Advanced growth modeling, survival analysis, and missing data analysis

Overview Of Mplus Courses (Continued)

- **Topic 5.** August, 2010, Johns Hopkins University: Categorical latent variable modeling with cross-sectional data
- **Topic 6.** August 2010, Johns Hopkins University: Categorical latent variable modeling with longitudinal data

- **Topic 7.** March, 2011, Johns Hopkins University: Multilevel modeling of cross-sectional data
- **Topic 8.** March 2011, Johns Hopkins University: Multilevel modeling of longitudinal data

Latent (Hidden) Markov Analysis

Latent (Hidden) Markov Analysis

Transition Probabilities

		c2	
		1	2
c1	1	0.8	0.2
	2	0.4	0.6

Transition Probabilities

For the c2 classes r = 1, 2, 3, the transition probabilities going from the classes of c1 to the classes of c2 are given by the following unordered multinomial logistic regression expressions:

$P\ (c2 = r \mid c1 = 1) = \exp\ (a_r + b_{r1}) / sum_1$,
$P\ (c2 = r \mid c1 = 2) = \exp\ (a_r + b_{r2}) / sum_2$,
$P\ (c2 = r \mid c1 = 3) = \exp\ (a_r + b_{r3}) / sum_3$,

where $a_3 = 0$, $b_{31} = 0$, $b_{32} = 0$, and $b_{33} = 0$ because the last class is the reference class, and sum_j represents the sum of the exponentiations across the classes of c2 for c1 = j (j = 1, 2, 3). The corresponding log odds when comparing a c2 class to the last c2 class are summarized in the table below.

		c2		
c1		1	2	3
	1	$a_1 + b_{11}$	$a_2 + b_{21}$	0
	2	$a_1 + b_{12}$	$a_2 + b_{22}$	0
	3	a_1	a_2	0

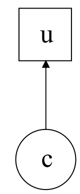
The parameters in the table are referred to in the MODEL command using the following statements:

a_1	[c2#1];
a_2	[c2#2];
b_{11}	c2#1 ON c1#1;
b_{12}	c2#1 ON c1#2;
b_{21}	c2#2 ON c1#1;
b_{22}	c2#2 ON c1#2;

Binary Latent Variable Measured By A Binary Indicator Without Error

***P* (*u* | c):**

		u = 0	u = 1	
c	1	1.0	0	1.0
	2	0	1.0	1.0



Mplus:

%c#1%
[u$1@15];

%c#2%
[u$1@-15];

Logits:

$P\ (u = 1 \mid c = 1) = 0$

$\rightarrow$ logit = $\log \left[\frac{0}{1-0} \right] = -\infty$

$\rightarrow$ threshold = $+\infty$ ($\approx + 15$)

(high threshold $\rightarrow$ low probability)

$P\ (u = 1 \mid c = 2) = 1 \rightarrow$

$\rightarrow$ logit = $\log \left[\frac{1}{1-1} \right] = +\infty$

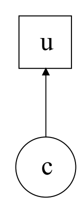
$\rightarrow$ threshold = $-\infty$ ($\approx - 15$)

(low threshold $\rightarrow$ high probability)

Binary Latent Variable Measured By A Binary Indicator With Error

***P* (*u* | c):**

		u = 0	u = 1	
c	1	0.7	0.3	1.0
	2	0.2	0.8	1.0



Mplus:

%c#1%
[u$1*0.847];

%c#2%
[u$1* – 1.386];

Logits:

$P\ (u = 1 \mid c = 1) = 0.3$

$\rightarrow$logit = $\log \left[\frac{0.3}{1-0.3} \right] = -0.847$

$\rightarrow$ threshold = $+ 0.847$

$P\ (u = 1 \mid c = 2) = 0.8$

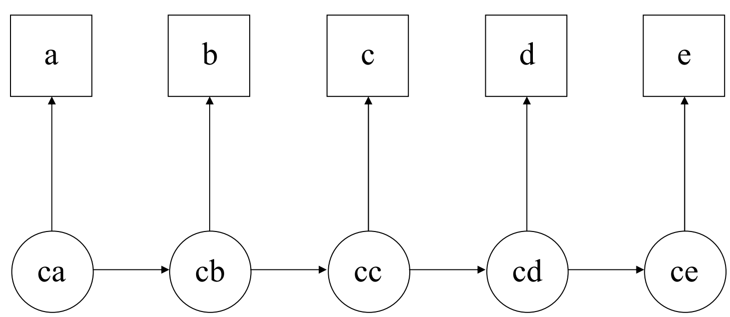
$\rightarrow$logit = $\log \left[\frac{0.8}{1-0.8} \right] = +1.386$

$\rightarrow$ threshold = $- 1.386$

Latent (Hidden) Markov Analysis
German Life Satisfaction

```
German Socio-Economic Panel
        n = 5147
        5 time points

"How satisfied are you on the whole with your life?"
        unsatisfied = 1
        satisfied = 2

Source: Langeheine & van de Pol (2002), chapter 13 of
Hagenaars & McCutcheon (2002), Applied Latent Class Analysis.
Data in Table 1, results in Table 3
```


Latent (Hidden) Markov Modeling



Input For Latent (Hidden) Markov Model

```
TITLE:     Latent (Hidden) Markov model in Table 3 of Langeheine &
           van de Pol (2002), chapter 13 of the book "Applied
           Latent Class Analysis", edited by Hagenaars &
           McCutcheon

DATA:      FILE =  chap11.dat;

VARIABLE:  NAMES = a b c d e male female count;
           MISSING = ALL (-9999);
           USEVARIABLES = a b c d e count;
           FREQWEIGHT = COUNT;
           CATEGORICAL = a b c d e;
           CLASSES = ca(2) cb(2) cc(2) cd(2) ce(2);
```


Input For Latent (Hidden) Markov Model (Continued)

```
ANALYSIS:  TYPE = MIXTURE;

MODEL:     %OVERALL%
           [cb#1 cc#1 cd#1 ce#1] (1);
           ce#1 ON cd#1 (2);
           cd#1 ON cc#1 (2);
           cc#1 ON cb#1 (2);
           cb#1 ON ca#1 (2);

MODEL ca:  %ca#1%
           [a$1]     (3);
           %ca#2%
           [a$1]     (4);
```

Input For Latent (Hidden) Markov Model (Continued)

```
MODEL cb:
           %cb#1%
           [b$1]     (3);
           %cb#2%
           [b$1]     (4);
MODEL cc:
           %cc#1%
           [c$1]     (3);
           %cc#2%
           [c$1]     (4);
MODEL cd:
           %cd#1%
           [d$1]     (3);
           %cd#2%
           [d$1]     (4);
MODEL ce:
           %ce#1%
           [e$1]     (3);
           %ce#2%
           [e$1]     (4);

OUTPUT:    TECH10;
```


Output Excerpts Latent (Hidden) Markov Model

Tests Of Model Fit

```
Loglikelihood
             H0 Value                        -15378.476

Information Criteria
             Number of Free Parameters                5
             Akaike (AIC)                     30766.952
             Bayesian (BIC)                   30799.683
             Sample-Sized Adjusted BIC        30783.795
              (n* = (n + 2)/24)
             Entropy                              0.777
```

Output Excerpts Latent (Hidden) Markov Model (Continued)

```
Chi-Square Test of Model Fit For the Binary and
Ordered Categorical (Ordinal) Outcomes

          Pearson Chi-Square

          Value                              243.841
          Degrees of Freedom                      26
          P-Value                             0.0000

          Likelihood Ratio Chi-Square

          Value                              235.928
          Degrees of Freedom                      26
          P-Value                             0.0000
```


Output Excerpts Latent (Hidden) Markov Model (Continued)

Estimated Observed Variable Proportions (TECH10)

```
UNIVARIATE MODEL FIT INFORMATION
                           Estimated Probabilities
    Variable             H1          H0      Standard Residual

     A
       Category 1       0.435       0.459         -3.511
       Category 2       0.565       0.541          3.511
     B
       Category 1       0.492       0.467          3.628
       Category 2       0.508       0.533         -3.628
     C
       Category 1       0.504       0.473          4.436
       Category 2       0.496       0.527         -4.436
     D
       Category 1       0.490       0.479          1.508
       Category 2       0.510       0.521         -1.508
     E
       Category 1       0.457       0.484         -3.856
       Category 2       0.543       0.516          3.856
```

Output Excerpts Latent (Hidden) Markov Model (Continued)

Estimated Latent Variable Proportions

FINAL CLASS COUNTS AND PROPORTIONS FOR EACH LATENT CLASS VARIABLE BASED ON THE ESTIMATED MODEL

Latent Class Variable	Class		
CA	1	2291.91235	0.44529
	2	2855.08765	0.55471
CB	1	2347.99268	0.45619
	2	2799.00781	0.54381
CC	1	2397.32251	0.46577
	2	2749.67773	0.53423
CD	1	2440.71509	0.47420
	2	2706.28516	0.52580
CE	1	2478.88477	0.48162
	2	2668.11499	0.51838

Output Excerpts Latent (Hidden) Markov Model (Continued)

Latent Transition Probabilities Based on the Estimated Model

CA Classes (Rows) by CB Classes (Columns)

	1	2
1	0.944	0.056
2	0.064	0.936

CB Classes (Rows) by CC Classes (Columns)

	1	2
1	0.944	0.056
2	0.064	0.936

CC Classes (Rows) by CD Classes (Columns)

	1	2
1	0.944	0.056
2	0.064	0.936

CD Classes (Rows) by CE Classes (Columns)

	1	2
1	0.944	0.056
2	0.064	0.936

Output Excerpts Latent (Hidden) Markov Model (Continued)

Results in Probability Scale

Latent Class Pattern 1 1 1 1 1

A			
Category 1	0.841	0.008	99.520
Category 2	0.159	0.008	18.803
B			
Category 1	0.841	0.008	99.520
Category 2	0.159	0.008	18.803
C			
Category 1	0.841	0.008	99.520
Category 2	0.159	0.008	18.803
D			
Category 1	0.841	0.008	99.520
Category 2	0.159	0.008	18.803
E			
Category 1	0.841	0.008	99.520
Category 2	0.159	0.008	18.803

Output Excerpts Latent (Hidden) Markov Model (Continued)

Results in Probability Scale

Latent Class Pattern 1 1 1 1 2

A			
Category 1	0.841	0.008	99.520
Category 2	0.159	0.008	18.803
B			
Category 1	0.841	0.008	99.520
Category 2	0.159	0.008	18.803
C			
Category 1	0.841	0.008	99.520
Category 2	0.159	0.008	18.803
D			
Category 1	0.841	0.008	99.520
Category 2	0.159	0.008	18.803
E			
Category 1	0.153	0.007	22.177
Category 2	0.847	0.007	117.549

Output Excerpts Latent (Hidden) Markov Model (Continued)

Model Fit Information for the Latent Class Indicator Model Part

RESPONSE PATTERNS

No.	Pattern	No.	Pattern	No.	Pattern	No.	Pattern
1	00000	2	10000	3	01000	4	11000
5	00100	6	10100	7	01100	8	11100
9	00010	10	10010	11	01010	12	11010
13	00110	14	10110	15	01110	16	11110
17	00001	18	10001	19	01001	20	11001
21	00101	22	10101	23	01101	24	11101
25	00011	26	10011	27	01011	28	11011
29	00111	30	10111	31	01111	32	11111

Output Excerpts Latent (Hidden) Markov Model (Continued)

Model Fit Information for the Latent Class Indicator Model Part

Response	Frequency		Standard	Chi-Square Contribution	
Pattern	Observed	Estimated	Residual	Pearson	Loglikelihood
1	891.00	793.27	3.77	12.04	1207.03
2	237.00	229.27	0.52	0.26	15.72
3	120.00	168.83	3.82	14.12	-81.93
4	136.00	127.24	0.79	0.60	18.11
5	111.00	159.30	3.89	14.64	-80.19
6	80.00	63.83	2.04	4.10	36.13
7	54.00	56.83	0.38	0.14	-5.52
8	99.00	130.50	2.79	7.60	-54.70
9	119.00	164.74	3.62	12.70	-77.41
10	68.00	54.55	1.83	3.32	29.97
11	51.00	44.00	1.06	1.11	15.05
12	64.00	67.35	0.41	0.17	-6.53
13	52.00	52.72	0.10	0.01	-1.43
14	51.00	57.36	0.84	0.70	-11.98
15	49.00	65.51	2.05	4.16	-28.46
16	172.00	256.95	5.44	28.08	-138.07

Output Excerpts Latent (Hidden) Markov Model (Continued)

Model Fit Information for the Latent Class Indicator Model Part

Response Pattern	Frequency Observed	Frequency Estimated	Standard Residual	Chi-Square Contribution Pearson	Chi-Square Contribution Loglikelihood
17	176.00	205.69	2.11	4.29	-54.87
18	107.00	64.40	5.34	28.18	108.65
19	64.00	50.16	1.96	3.82	31.19
20	95.00	62.22	4.18	17.27	80.41
21	60.00	55.33	0.63	0.39	9.73
22	75.00	48.04	3.91	15.12	66.80
23	50.00	53.09	0.43	0.18	-6.00
24	165.00	197.97	2.39	5.49	-60.12
25	106.00	99.40	0.67	0.44	13.62
26	107.00	57.93	6.48	41.55	131.29
27	67.00	58.80	1.08	1.14	17.50
28	187.00	188.24	0.09	0.01	-2.47
29	92.00	102.64	1.06	1.10	-20.13
30	200.00	193.66	0.46	0.21	12.88
31	176.00	233.22	3.83	14.04	-99.09
32	1066.00	983.95	2.91	6.84	170.76

Further Readings On Markov Modeling

Langeheine, R. & van de Pol, F. (2002). Latent Markov chains. In Hagenaars, J.A. & McCutcheon, A.L. (eds.), Applied latent class analysis (pp. 304-341). Cambridge, UK: Cambridge University Press.

Mooijaart, A. (1998). Log-linear and Markov modeling of categorical longitudinal data. In Bijleveld, C. C. J. H., & van der Kamp, T. (eds). Longitudinal data analysis: Designs, models, and methods. Newbury Park: Sage.

Latent Transition Analysis

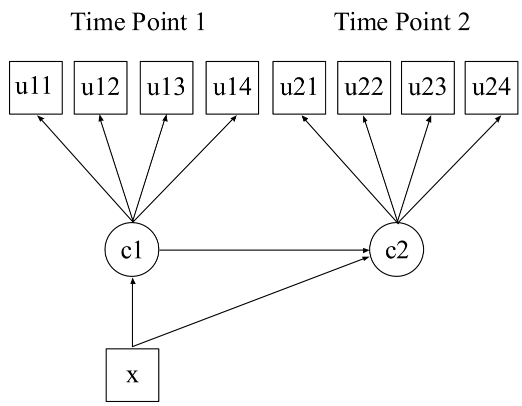
Latent Transition Analysis

- Setting
  - Cross-sectional or longitudinal data
  - Multiple items measuring several different constructs
  - Hypothesized simple structure for measurements
  - Hypothesized constructs represented as latent class variables (categorical latent variables)
- Aim
  - Identify items that indicate classes well
  - Test simple measurement structure
  - Study relationships between latent class variables
  - Estimate class probabilities
  - Relate class probabilities to covariates
  - Classify individuals into classes (posterior probabilities)
- Application
  - Latent transition analysis with four latent class indicators at two time points and a covariate

Latent Transition Analysis

Transition Probabilities

c1 \ c2	1	2
1	0.8	0.2
2	0.4	0.6



Input For LTA With Two Time Points And A Covariate

```
TITLE:      Latent transition analysis for two time points and a
            covariate

DATA:       FILE = mc2tx.dat;

VARIABLE:   NAMES ARE u11-u14 u21-u24 x xc1 xc2;
            USEV = u11-u14 u21-u24 x;
            CATEGORICAL = u11-u24;
            CLASSES = c1(2) c2(2);

ANALYSIS:   TYPE = MIXTURE;

MODEL:      %OVERALL%
            c2#1 ON c1#1 x;
            c1#1 ON x;
```

Input For LTA With Two Time Points And A Covariate (Continued)

```
MODEL c1:
           %c1#1%
           [u11$1-u14$1] (1-4);
           %c1#2%
           [u11$1-u14$1] (5-8);

MODEL c2:
           %c2#1%
           [u21$1-u24$1] (1-4);
           %c2#2%
           [u21$1-u24$1] (5-8);

OUTPUT: TECH1 TECH8;
```


Output Excerpts LTA With Two Time Points And A Covariate

Tests Of Model Fit

```
Loglikelihood
           H0 Value                     -3926.187
Information Criteria
           Number of Free Parameters           13
           Akaike (AIC)                  7878.374
           Bayesian (BIC)                7942.175
           Sample-Size Adjusted BIC      7900.886
              (n* = (n + 2) / 24)
           Entropy                          0.902
```

Output Excerpts LTA With Two Time Points And A Covariate (Continued)

```
Chi-Square Test of Model Fit for the Latent Class Indicator Model
Part
            Pearson Chi-Square

            Value                           250.298
            Degrees of Freedom                  244
            P-Value                          0.3772

            Likelihood Ratio Chi-Square

            Value                           240.811
            Degrees of Freedom                  244
            P-Value                          0.5457
```

Final Class Counts

```
FINAL  CLASS COUNTS AND PROPORTIONS OF TOTAL SAMPLE SIZE BASED
ON ESTIMATED POSTERIOR PROBABILITIES
 Class 1         328.42644          0.32843
 Class 2         184.43980          0.18444
 Class 3         146.98726          0.14699
 Class 4         340.14650          0.34015
```


Output Excerpts LTA With Two Time Points And A Covariate (Continued)

Model Results

```
                    Estimates      S.E.   Est./S.E.

Class 1-C1, 1-C2

  Thresholds
     U11$1              -2.020      0.110    -18.353
     U12$1              -2.003      0.106    -18.919
     U13$1              -1.776      0.098    -18.046
     U14$1              -1.861      0.101    -18.396
     U21$1              -2.020      0.110    -18.353
     U22$1              -2.003      0.106    -18.919
     U23$1              -1.776      0.098    -18.046
     U24$1              -1.861      0.101    -18.396
```

Output Excerpts LTA With Two Time Points And A Covariate (Continued)

Class 1-C1, 2-C2

Thresholds

U11$1	-2.020	0.110	-18.353
U12$1	-2.003	0.106	-18.919
U13$1	-1.776	0.098	-18.046
U14$1	-1.861	0.101	-18.396
U21$1	1.964	0.111	17.736
U22$1	2.164	0.119	18.113
U23$1	1.864	0.100	18.704
U24$1	2.107	0.112	18.879

Class 2-C1, 1-C2

Thresholds

U11$1	1.964	0.111	17.736
U12$1	2.164	0.119	18.113
U13$1	1.864	0.100	18.704
U14$1	2.107	0.112	18.879
U21$1	-2.020	0.110	-18.353
U22$1	-2.003	0.106	-18.919
U23$1	-1.776	0.098	-18.046
U24$1	-1.861	0.101	-18.396

Output Excerpts LTA With Two Time Points And A Covariate (Continued)

Class 2-C1, 2-C2

Thresholds

	Estimates	S.E.	Est./S.E.
U11$1	1.964	0.111	17.736
U12$1	2.164	0.119	18.113
U13$1	1.864	0.100	18.704
U14$1	2.107	0.112	18.879
U21$1	1.964	0.111	17.736
U22$1	2.164	0.119	18.113
U23$1	1.864	0.100	18.704
U24$1	2.107	0.112	18.879

Output Excerpts LTA With Two Time Points And A Covariate (Continued)

	Estimates	S.E.	Est./S.E.
C2#1 ON			
C1#1	0.530	0.180	2.953
C2#1 ON			
X	-1.038	0.107	-9.703
C1#1 ON			
X	-1.540	0.112	-13.761
Intercepts			
C1#1	0.065	0.082	0.797
C2#1	-0.407	0.120	-3.381

Steps In Latent Transition Analysis

- Step 1: Study measurement model alternatives for each time point
- Step 2: Explore transitions based on cross-sectional results
  - Cross-tabs based on most likely class membership
- Step 3: Explore specification of the latent transition model without covariates
  - Testing for measurement invariance across time
- Step 4: Include covariates in the LTA model
- Step 5: Include distal outcomes and advanced modeling extensions

Source: Nylund (2007)

Mover-Stayer Latent Transition Analysis

Mover-Stayer Latent Transition Analysis

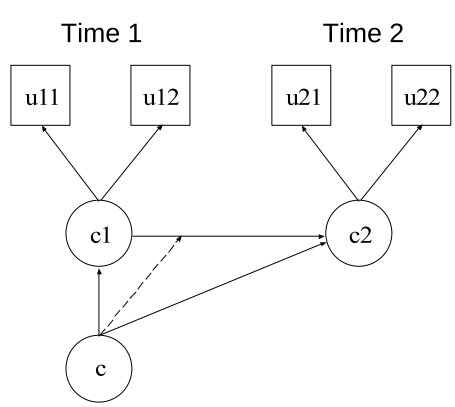
Transition Probabilities

Mover Class (c = 1)

c1 \ c2	1	2
1	0.6	0.4
2	0.3	0.7

Stayer Class (c = 2)

c1 \ c2	1	2
1	0.90	0.10
2	0.05	0.95



Further Readings On Latent Transition Analysis

Chung, H., Park, Y., & Lanza, S.T. (2005). Latent transition analysis with covariates: pubertal timing and substance use behaviors in adolescent females. Statistics in Medicine, 24, 2895 - 2910.

Collins, L.M. & Wugalter, S.E. (1992). Latent class models for stage-sequential dynamic latent variables. Multivariate Behavioral Research, 27, 131-157.

Collins, L.M., Graham, J.W., Rousculp, S.S., & Hansen, W.B. (1997). Heavy caffeine use and the beginning of the substance use onset process: An illustration of latent transition analysis. In K. Bryant, M. Windle, & S. West (Eds.), The science of prevention: Methodological advances from alcohol and substance use research. Washington DC: American Psychological Association. pp. 79-99.

Kaplan, D. (2008). An overview of Markov chain methods for the study of stage-sequential developmental processes. Developmental Psychology, 44, 457-467.

Further Readings On Latent Transition Analysis (Continued)

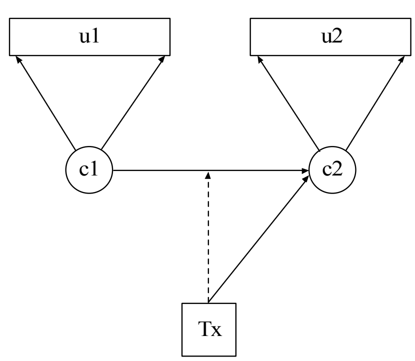
Nylund, K.L., Muthén, B., Nishina, A., Bellmore, A. & Graham, S. (2006). Stability and instability of peer victimization during middle school: Using latent transition analysis with covariates, distal outcomes, and modeling extensions. Submitted for publication.

Nylund, K. (2007). Latent transition analysis: Modeling extensions and an application to peer victimization. Doctoral dissertation, University of California, Los Angeles.

Reboussin, B.A., Reboussin, D,M., Liang, K.Y., & Anthony, J.C. (1998). Latent transition modeling of progression of health-risk behavior. Multivariate Behavioral Research, 33, 457-478.

Latent Transition Analysis Extensions

Latent Transition Analysis
And Intervention Studies



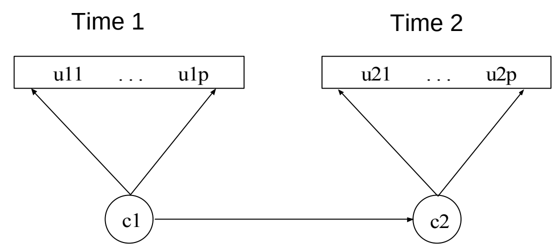
Input For LTA With An Intervention

```
VARIABLE:
        NAMES = u11-u15 u21-u25 tx;
        CATEGORICAL = u11-u15 u21-u25;
        CLASSES = cg(2) c1(2) c2(2);
        KNOWNCLASS = cg(tx=0 tx=1);
ANALYSIS:
        TYPE = MIXTURE;
        PROCESS = 2;
        STARTS = 100 20;
MODEL:
        %OVERALL%
        c2#1 ON c1#1@0
                cg#1 (p0);
        [c2#1] (p1);
MODEL cg:
        %cg#1%
        c2#1 ON c1#1 (p2);
        %cg#2%
        c2#1 ON c1#1 (p3);
MODEL c1:
        %c1#1%
        [u11$1-u15$1*1] (1-5);
        %c1#2%
        [u11$1-u15$1*-1] (11-15);
```



```
MODEL c2:
        %c2#1%
        [u21$1-u25$1*1] (1-5);

        %c2#2%
        [u21$1-u25$1*-1] (11-15);

MODEL CONSTRAINT:
        NEW(p011 p012 p021 p022 p111 p112 p121 p122 lowlow highlow);
        !p0*, p1* contain probabilities for the 4 cells for control and
        !intervention groups
        !lowlow is the probability effect of intervention on staying in
        !the low class
        !highlow is the probability effect of intervention on moving from
        !high to low class
        !the effect is calculated as P(intervention)-P(control)
        p011 = exp(p0+p1+p2)/(exp(p0+p1+p2)+1);
        p012 = 1/(exp(p0+p1+p2)+1);
        p021 = exp(p0+p1)/(exp(p0+p1)+1);
        p022 = 1/(exp(p0+p1)+1);
        p111 = exp(p1+p3)/(exp(p1+p3)+1);
        p112 = 1/(exp(p1+p3)+1);
        p121 = exp(p1)/(exp(p1)+1);
        p122 = 1/(exp(p1)+1);
        lowlow = p111-p011;
        highlow = p121-p021;
OUTPUT:
        TECH1 PATTERNS;
PLOT:
        TYPE = PLOT3;
```

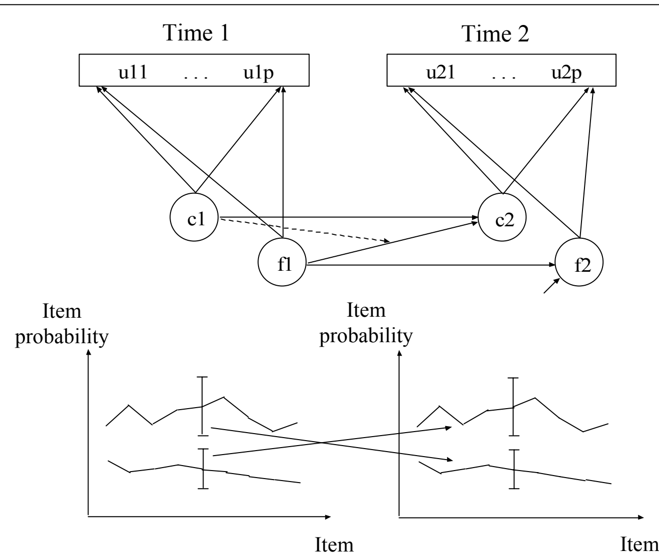
Latent Transition Analysis



Transition Probabilities

		c2: 1	c2: 2
c1	1	0.6	0.4
	2	0.3	0.7

Factor Mixture Latent Transition Analysis Muthen (2006)



Factor Mixture Latent Transition Analysis: Aggressive-Disruptive Behavior In The Classroom

- 1,137 first-grade students in Baltimore public schools
- 9 items: Stubborn, Break rules, Break things, Yells at others, Takes others property, Fights, Lies, Teases classmates, Talks back to adults
- Skewed, 6-category items; dichotomized (almost never vs other)
- Two time points: Fall and Spring of Grade 1
- For each time point, a 2-class, 1-factor FMA was found best fitting

Factor Mixture Latent Transition Analysis: Aggressive-Disruptive Behavior In The Classroom (Continued)

Model	Loglikelihood	# parameters	BIC
Conventional LTA	-8,649	21	17,445
FMA LTA factors related across time	-8,012	40	16,306

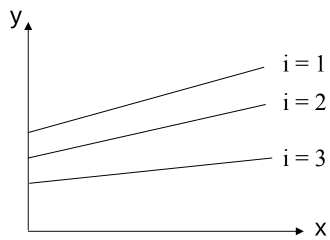
Factor Mixture Latent Transition Analysis: Aggressive-Disruptive Behavior In The Classroom (Continued)

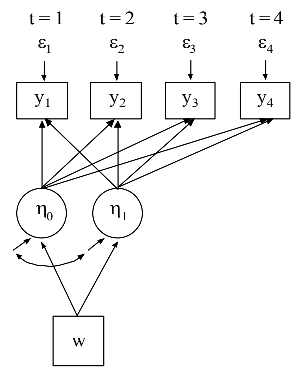
Estimated Latent Transition Probabilities, Fall to Spring

Conventional LTA		
	Low	High
Low	0.93	0.07
High	0.17	0.83
FMA-LTA		
	Low	High
Low	0.94	0.06
High	**0.41**	**0.59**

Growth Modeling With Random Effects

Individual Development Over Time



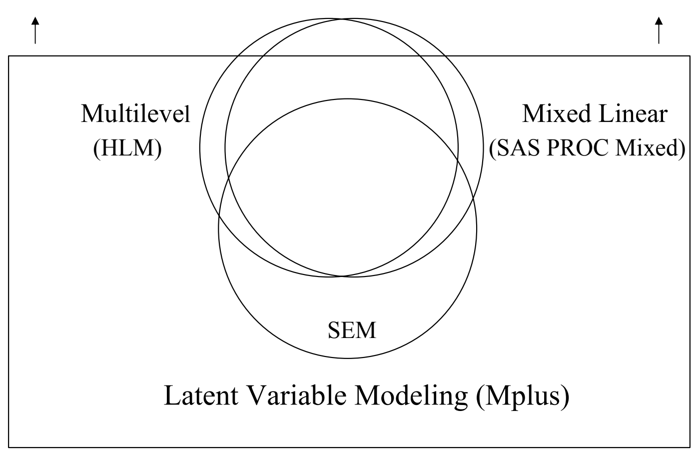


(1) $y_{ti} = \eta_{0i} + \eta_{1i}\, x_t + \varepsilon_{ti}$

(2a) $\eta_{0i} = \alpha_0 + \gamma_0\, w_i + \zeta_{0i}$

(2b) $\eta_{1i} = \alpha_1 + \gamma_1\, w_i + \zeta_{1i}$

Growth Modeling Frameworks/Software



Advantages Of Growth Modeling In A Latent Variable Framework

- Flexible curve shape
- Individually-varying times of observation
- Regressions among random effects
- Multiple processes
- Modeling of zeroes
- Multiple populations
- Multiple indicators
- Embedded growth models
- Categorical latent variables: growth mixtures

Growth Models With Categorical Outcomes

Growth Model With Categorical Outcomes

- Individual differences in development of probabilities over time
- Logistic model considers growth in terms of log odds (logits), e.g.

(1) $$\log\left[\frac{P(u_{ti}=1 \mid \eta_{0i}, \eta_{1i}, \eta_{2i}, x_{ti})}{P(u_{ti}=0 \mid \eta_{0i}, \eta_{1i}, \eta_{2i}, x_{ti})}\right] = \eta_{0i} + \eta_{1i} \cdot (x_{ti} - c) + \eta_{2i} \cdot (x_{ti} - c)^2$$

for a binary outcome using a quadratic model with centering at time c. The growth factors η_{0i}, η_{1i}, and η_{2i} are assumed multivariate normal given covariates,

(2a) $\eta_{0i} = \alpha_0 + \gamma_0 w_i + \zeta_{0i}$

(2b) $\eta_{1i} = \alpha_1 + \gamma_1 w_i + \zeta_{1i}$

(2c) $\eta_{2i} = \alpha_2 + \gamma_2 w_i + \zeta_{2i}$

Growth Models With Categorical Outcomes

- Measurement invariance of the outcome over time is represented by the equality of thresholds over time (rather than intercepts)

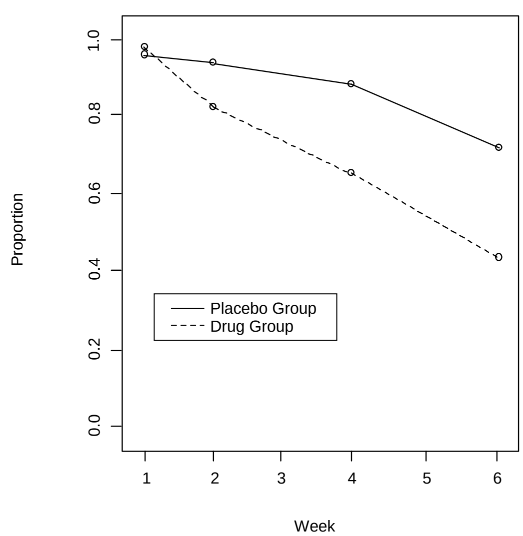
- Thresholds not set to zero but held equal across timepoints—intercept factor mean value fixed at zero

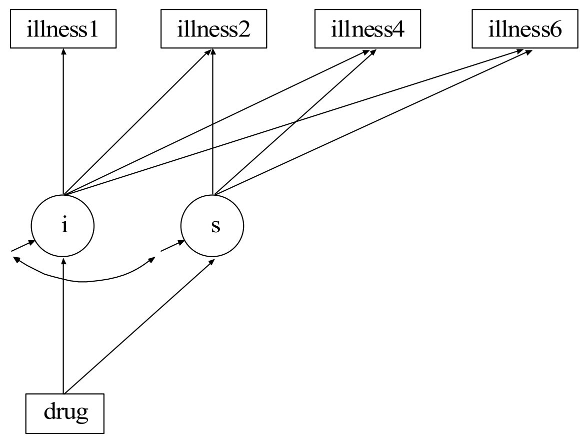
- Differences in variances of the outcome over time are represented by allowing scale parameters to vary over time

The NIMH Schizophrenia Collaborative Study

- The Data—The NIMH Schizophrenia Collaborative Study (Schizophrenia Data)
  - A group of 64 patients using a placebo and 249 patients on a drug for schizophrenia measured at baseline and at weeks one through six
  - Variables—severity of illness, background variables, and treatment variable
- Data for the analysis—severity of illness at weeks one, two, four, and six and treatment

Schizophrenia Data: Sample Proportions





Input For Schizophrenia Data Growth Model For Binary Outcomes With A Treatment Variable

```
TITLE:      Schizophrenia Data
            Growth Model for Binary Outcomes
            With a Treatment Variable and Scaling Factors

DATA:       FILE IS schiz.dat; FORMAT IS 5F1;

VARIABLE:   NAMES ARE illness1 illness2 illness4 illness6
            drug;   ! 0=placebo (n=64)  1=drug (n=249)
            CATEGORICAL ARE illness1-illness6;

ANALYSIS:   ESTIMATOR = ML;
            !ESTIMATOR = WLSMV;

MODEL:      i s | illness1@0 illness2@1 illness4@3
                  illness6@5;
            i s ON drug;
```

Alternative language:

```
MODEL:      i BY illness1-illness6@1;
            s BY illness1@0 illness2@1 illness4@3 illness6@5;
            [illness1$1 illness2$1 illness4$1 illness6$1] (1);
            [s];
            i s ON drug;
            !{illness1@1 illness2-illness6};
```

Output Excerpts Schizophrenia Data Growth Model For Binary Outcomes With A Treatment Variable

n = 313

Tests Of Model Fit

```
Loglikelihood
          HO Value                     -486.337
Information Criteria
          Number of Free Parameters           7
          Akaike (AIC)                   986.674
          Bayesian (BIC)                1012.898
          Sample-Size Adjusted BIC       990.696
             (n* = (n + 2) / 24)
```


Output Excerpts Schizophrenia Data Growth Model For Binary Outcomes With A Treatment Variable (Continued)

Selected Estimates

	Estimates	S.E.	Est./S.E.	Std	StdYX
I ON					
DRUG	-0.429	0.825	-0.521	-0.156	-0.063
S ON					
DRUG	-0.651	0.259	-2.512	-0.684	-0.276
I WITH					
S	-0.925	0.621	-1.489	-0.353	-0.353
Intercepts					
I	0.000	0.000	0.000	0.000	0.000
S	-0.555	0.255	-2.182	-0.583	-0.583
Thresholds					
ILLNESS1$1	-5.706	1.047	-5.451		
ILLNESS2$1	-5.706	1.047	-5.451		
ILLNESS4$1	-5.706	1.047	-5.451		
ILLNESS5$1	-5.706	1.047	-5.451		
Residual Variances					
I	7.543	3.213	2.348	0.996	0.996
S	0.838	0.343	2.440	0.924	0.924

Further Readings On Growth Analysis With Categorical Outcomes

Fitzmaurice, G.M., Laird, N.M. & Ware, J.H. (2004). Applied longitudinal analysis. New York: Wiley.

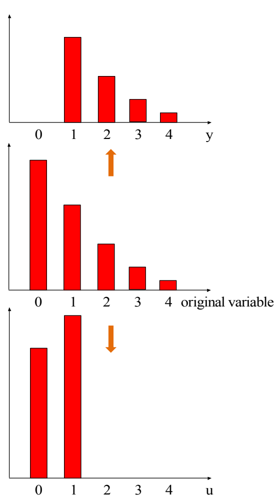
Gibbons, R.D. & Hedeker, D. (1997). Random effects probit and logistic regression models for three-level data. Biometrics, 53, 1527-1537.

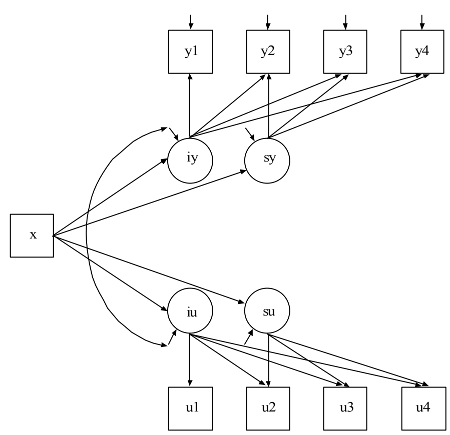
Hedeker, D. & Gibbons, R.D. (1994). A random-effects ordinal regression model for multilevel analysis. Biometrics, 50, 933-944.

Muthén, B. (1996). Growth modeling with binary responses. In A. V. Eye, & C. Clogg (Eds.), Categorical variables in developmental research: methods of analysis (pp. 37-54). San Diego, CA: Academic Press. (#64)

Muthén, B. & Asparouhov, T. (2002). Latent variable analysis with categorical outcomes: Multiple-group and growth modeling in Mplus. Mplus Web Note #4 (www.statmodel.com).

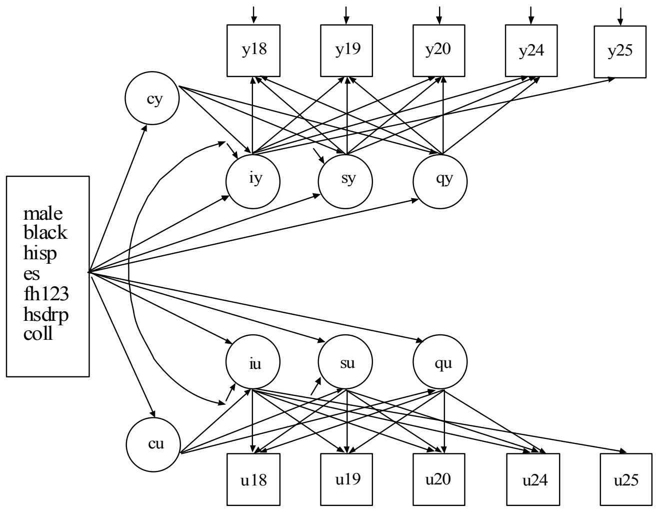
Two-Part Growth Modeling: Frequency Of Heavy Drinking Ages 18 – 25





Olsen and Schafer (2001)

Two-Part Growth Mixture Modeling



Two-Part Modeling Extensions In Mplus

- Growth modeling
  - Distal outcome
  - Parallel processes
  - Trajectory classes (mixtures)
  - Multilevel
- Factor analysis
  - Mixtures (Kim & Muthen, 2007)
    - Latent classes for binary and continuous parts may be incorrectly picked up as additional factors in conventional analysis
  - Multilevel

Latent Class Growth Analysis

Latent Class Growth Analysis

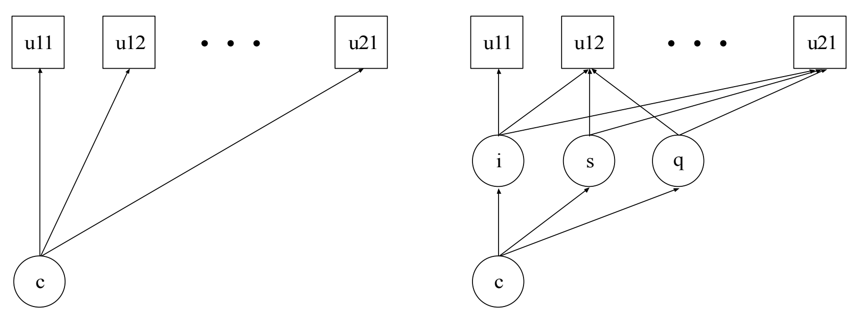
- Setting
  - Longitudinal data
  - A single item measured repeatedly
  - Hypothesized trajectory classes (categorical latent variable)
- Aim
  - Estimate trajectory shapes
  - Estimate trajectory class probabilities
  - Relate class probabilities to covariates
  - Classify individuals into classes (posterior probabilities)
- Applications
  - Single process
  - Two processes

Single Process Latent Class Growth Analysis: Cambridge Delinquency Data

- 411 boys in a working class section of London (n = 403 due to 8 boys who died)
- Ages 10 to 32 (ages 11 - 21 used here)
- Outcome is number of convictions in the last 2 years, modeled as an ordered polytomous variable scored 0 for 0 convictions, 1 for one conviction, and 2 for more than one conviction

Sources: Farrington & West (1990); Nagin & Land (1993); Roeder, Lynch & Nagin (1999); Muthen (2004)

LCA Vs LCGA



Number of parameters (11 *u*'s, 3 classes):

	LCA	LCGA
binary *u*	35	11
3-categ. *u*	68	12

Latent Class Analysis With 3 Classes On Cambridge Data

LogL = -1,032 (68 parameters), BIC = 2,472

Input LCGA On Cambridge Data

```
TITLE:      LCGA
            ordered polytomous variables for conviction at each
            age11-21
            dep. variable 0, 1 , 2 (0, 1, or more convictions)

DATA:       FILE = naginordered.dat;

VARIABLE:   NAMES = u11 u12 u13 u14 u15 u16 u17 u18 u19 u20
                    u21 c1 c2 c3 c4;
            USEVAR = u11-u21;
            CATEGORICAL = u11-u21;
            CLASSES = c(3);

ANALYSIS:   TYPE = MIXTURE;
```

Input LCGA On Cambridge Data (Continued)

```
MODEL:      %OVERALL%

            i s q | u11@-.6  u12@-.5   u13@-.4  u14@-.3  u15@-.2
            u16@-.1  u17@0  u18@.1  u19@.2  u20@.3  u21@.4;

OUTPUT:     TECH1 TECH8;

PLOT:       SERIES = u11-u21(s);
            TYPE = PLOT3;
```


Output Excerpts LCGA On Cambridge Data

Model Results

```
                        Estimates      S.E.     Est./S.E.
Latent Class 1

I   |
  u11                       1.000     0.000        0.000
  u12                       1.000     0.000        0.000
  u13                       1.000     0.000        0.000
  u14                       1.000     0.000        0.000
  u15                       1.000     0.000        0.000
  u16                       1.000     0.000        0.000
  u17                       1.000     0.000        0.000
  u18                       1.000     0.000        0.000
  u19                       1.000     0.000        0.000
  u20                       1.000     0.000        0.000
  u21                       1.000     0.000        0.000
```

Output Excerpts LCGA On Cambridge Data (Continued)

```
                    Estimates       S.E.    Est./S.E.
Latent Class 1

S  |
  u11                  -0.600      0.000        0.000
  u12                  -0.500      0.000        0.000
  u13                  -0.400      0.000        0.000
  u14                  -0.300      0.000        0.000
  u15                  -0.200      0.000        0.000
  u16                  -0.100      0.000        0.000
  u17                   0.000      0.000        0.000
  u18                   0.100      0.000        0.000
  u19                   0.200      0.000        0.000
  u20                   0.300      0.000        0.000
  u21                   0.400      0.000        0.000
```


Output Excerpts LCGA On Cambridge Data (Continued)

```
                    Estimates       S.E.    Est./S.E.
Latent Class 1

Q  |
  u11                   0.360      0.000        0.000
  u12                   0.250      0.000        0.000
  u13                   0.160      0.000        0.000
  u14                   0.090      0.000        0.000
  u15                   0.040      0.000        0.000
  u16                   0.010      0.000        0.000
  u17                   0.000      0.000        0.000
  u18                   0.010      0.000        0.000
  u19                   0.040      0.000        0.000
  u20                   0.090      0.000        0.000
  u21                   0.160      0.000        0.000

Means
I                      -1.633      0.329       -4.955
S                       0.264      0.431        0.613
Q                      -7.376      0.249       -5.906
```

Output Excerpts LCGA On Cambridge Data (Continued)

	Estimates	S.E.	Est./S.E.
Thresholds			
u11$1	-0.917	0.319	-2.876
u11$2	0.827	0.304	2.716
u12$1	-0.917	0.319	-2.876
u12$2	0.827	0.304	2.716
u13$1	-0.917	0.319	-2.876
u13$2	0.827	0.304	2.716
u14$1	-0.917	0.319	-2.876
u14$2	0.827	0.304	2.716
u15$1	-0.917	0.319	-2.876
u15$2	0.827	0.304	2.716
u16$1	-0.917	0.319	-2.876
u16$2	0.827	0.304	2.716
u17$1	-0.917	0.319	-2.876
u17$2	0.827	0.304	2.716
u18$1	-0.917	0.319	-2.876
u18$2	0.827	0.304	2.716
u19$1	-0.917	0.319	-2.876
u19$2	0.827	0.304	2.716
u20$1	-0.917	0.319	-2.876
u20$2	0.827	0.304	2.716
u21$1	-0.917	0.319	-2.876
u21$2	0.827	0.304	2.716

Output Excerpts LCGA On Cambridge Data (Continued)

	Estimates	S.E.	Est./S.E.
Latent Class 2			
Q |			
u11	0.360	0.000	0.000
u12	0.250	0.000	0.000
u13	0.160	0.000	0.000
u14	0.090	0.000	0.000
u15	0.040	0.000	0.000
u16	0.010	0.000	0.000
u17	0.000	0.000	0.000
u18	0.010	0.000	0.000
u19	0.040	0.000	0.000
u20	0.090	0.000	0.000
u21	0.160	0.000	0.000
Means			
I	-5.246	0.509	-10.297
S	0.802	0.836	0.959
Q	-2.796	2.515	-1.112

Output Excerpts LCGA On Cambridge Data (Continued)

	Estimates	S.E.	Est./S.E.
Thresholds			
u11$1	-0.917	0.319	-2.876
u11$2	0.827	0.304	2.716
u12$1	-0.917	0.319	-2.876
u12$2	0.827	0.304	2.716
u13$1	-0.917	0.319	-2.876
u13$2	0.827	0.304	2.716
u14$1	-0.917	0.319	-2.876
u14$2	0.827	0.304	2.716
u15$1	-0.917	0.319	-2.876
u15$2	0.827	0.304	2.716
u16$1	-0.917	0.319	-2.876
u16$2	0.827	0.304	2.716
u17$1	-0.917	0.319	-2.876
u17$2	0.827	0.304	2.716
u18$1	-0.917	0.319	-2.876
u18$2	0.827	0.304	2.716
u19$1	-0.917	0.319	-2.876
u19$2	0.827	0.304	2.716
u20$1	-0.917	0.319	-2.876
u20$2	0.827	0.304	2.716
u21$1	-0.917	0.319	-2.876
u21$2	0.827	0.304	2.716

Output Excerpts LCGA On Cambridge Data (Continued)

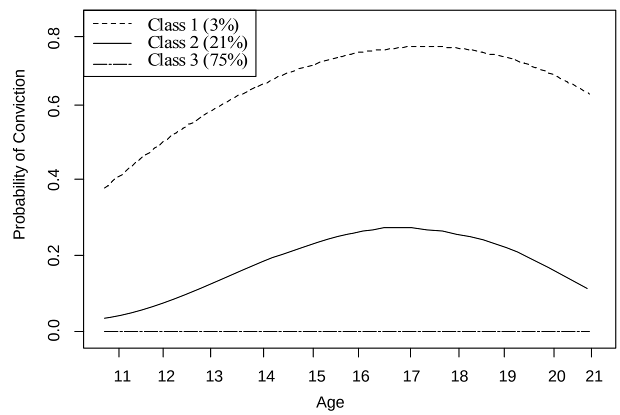
	Estimates	S.E.	Est./S.E.
Latent Class 3			
Q |			
u11	0.360	0.000	0.000
u12	0.250	0.000	0.000
u13	0.160	0.000	0.000
u14	0.090	0.000	0.000
u15	0.040	0.000	0.000
u16	0.010	0.000	0.000
u17	0.000	0.000	0.000
u18	0.010	0.000	0.000
u19	0.040	0.000	0.000
u20	0.090	0.000	0.000
u21	0.160	0.000	0.000
Means			
I	0.000	0.000	0.000
S	0.311	1.012	0.308
Q	-3.853	0.943	-1.983

Output Excerpts LCGA On Cambridge Data (Continued)

```
                       Estimates      S.E.    Est./S.E.
Thresholds
  u11$1                   -0.917      0.319      -2.876
  u11$2                    0.827      0.304       2.716
  u12$1                   -0.917      0.319      -2.876
  u12$2                    0.827      0.304       2.716
  u13$1                   -0.917      0.319      -2.876
  u13$2                    0.827      0.304       2.716
  u14$1                   -0.917      0.319      -2.876
  u14$2                    0.827      0.304       2.716
  u15$1                   -0.917      0.319      -2.876
  u15$2                    0.827      0.304       2.716
  u16$1                   -0.917      0.319      -2.876
  u16$2                    0.827      0.304       2.716
  u17$1                   -0.917      0.319      -2.876
  u17$2                    0.827      0.304       2.716
  u18$1                   -0.917      0.319      -2.876
  u18$2                    0.827      0.304       2.716
  u19$1                   -0.917      0.319      -2.876
  u19$2                    0.827      0.304       2.716
  u20$1                   -0.917      0.319      -2.876
  u20$2                    0.827      0.304       2.716
  u21$1                   -0.917      0.319      -2.876
  u21$2                    0.827      0.304       2.716
```


LCGA On Cambridge Data (Continued)

3-class LCGA
LogL = -1,072
(12 parameters)
BIC = 2,215



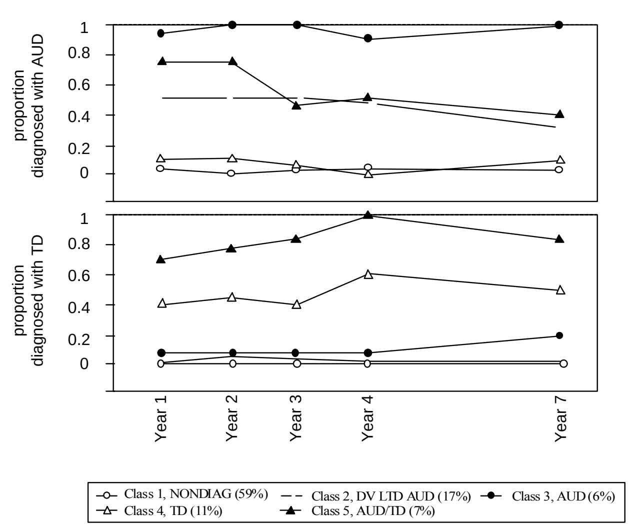
3-class LCA
LogL = -1,032
(68 parameters)
BIC = 2,472

Multiple Process LCGA: Relating Trajectory Class Variables For Different Outcomes

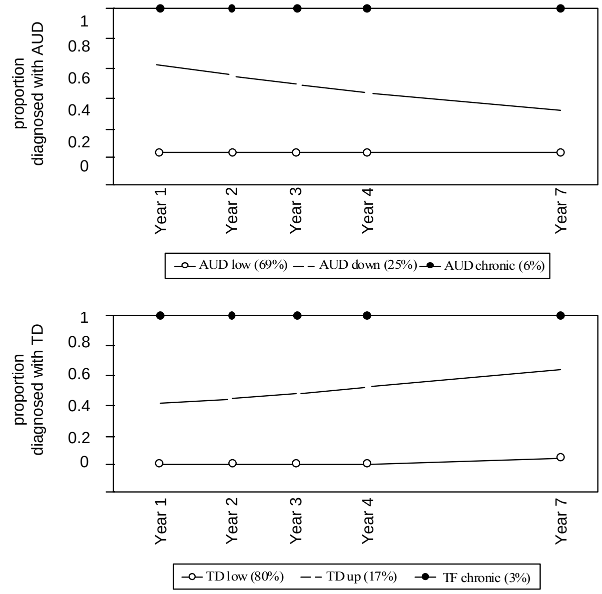
The co-occurrence of alcohol and tobacco use disorders (Jackson, Sher, Wood, 1999)

- Parallel processes
- College sample, n = 450

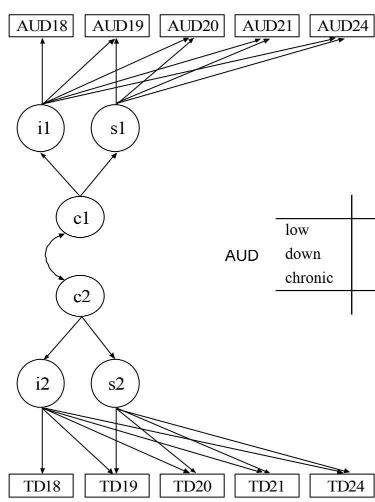
Co-Occurrence Of Alcohol And Tobacco Use Disorder



Co-Occurrence Of Alcohol And Tobacco Use Disorder



Co-Occurrence Of Alcohol And Tobacco Use Disorder



Class Probability Estimates

		TD			
		low	up	chronic	
AUD	low	.61	.08	.001	.69
	down	.15	.07	.03	.25
	chronic	.04	.02	.003	.06
		.80	.17	.03	1.00

Further Readings On Latent Class Growth Analysis

Jones, B.L., Nagin, D.S. & Roeder, K. (2001). A SAS procedure based on mixture models for estimating developmental trajectories. Sociological Methods & Research, 29, 374-393.

Land, K.C. (2001). Introduction to the special issue on finite mixture models. Sociological Methods & Research, 29, 275-281.

Muthén, B. (2001). Latent variable mixture modeling. In G. A. Marcoulides & R. E. Schumacker (eds.), New developments and techniques in structural equation modeling (pp. 1-33). Lawrence Erlbaum Associates. (#86)

Nagin, D.S. (1999). Analyzing developmental trajectories: a semi-parametric, group-based approach. Psychological Methods, 4, 139-157.

Nagin, D.S. (2005). Group-based modeling of development. Cambridge: Harvard University Press.

Further Readings On Latent Class Growth Analysis (Continued)

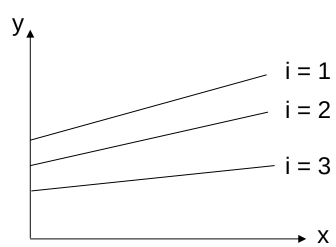
Nagin, D.S. & Land, K.C. (1993). Age, criminal careers, and population heterogeneity: Specification and estimation of a nonparametric, mixed Poisson model. Criminology, 31, 327-362.

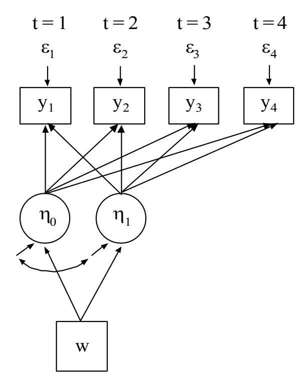
Nagin, D.S. & Tremblay, R.E. (1999). Trajectories of boys' physical aggression, opposition, and hyperactivity on the path to physically violent and non violent juvenile delinquency. Child Development, 70, 1181-1196.

Nagin, D.S. & Tremblay, R.E. (2001). Analyzing developmental trajectories of distinct but related behaviors: A group-based method. Psychological Methods, 6, 18-34.

Growth Mixture Modeling

Individual Development Over Time





(1) $y_{ti} = \eta_{0i} + \eta_{1i}\, x_t + \varepsilon_{ti}$

(2a) $\eta_{0i} = \alpha_0 + \gamma_0\, w_i + \zeta_{0i}$

(2b) $\eta_{1i} = \alpha_1 + \gamma_1\, w_i + \zeta_{1i}$

Mixtures And Latent Trajectory Classes

Modeling motivated by substantive theories of:

- Multiple Disease Processes: Prostate cancer (Pearson et al.)
- Multiple Pathways of Development: Adolescent-limited versus life-course persistent antisocial behavior (Moffitt), crime curves (Nagin), alcohol development (Zucker, Schulenberg)
- Subtypes: Subtypes of alcoholism (Cloninger, Zucker)

Example: Mixed-Effects Regression Models For Studying The Natural History Of Prostate Disease

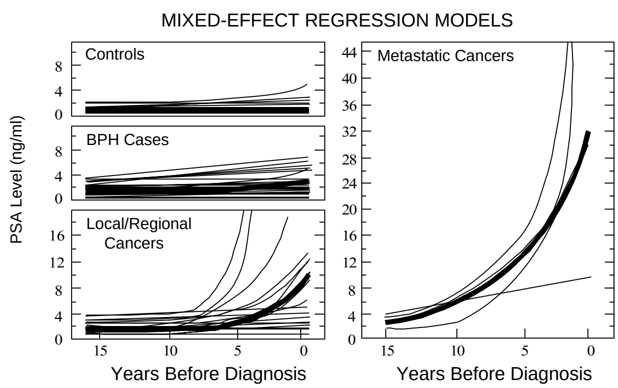
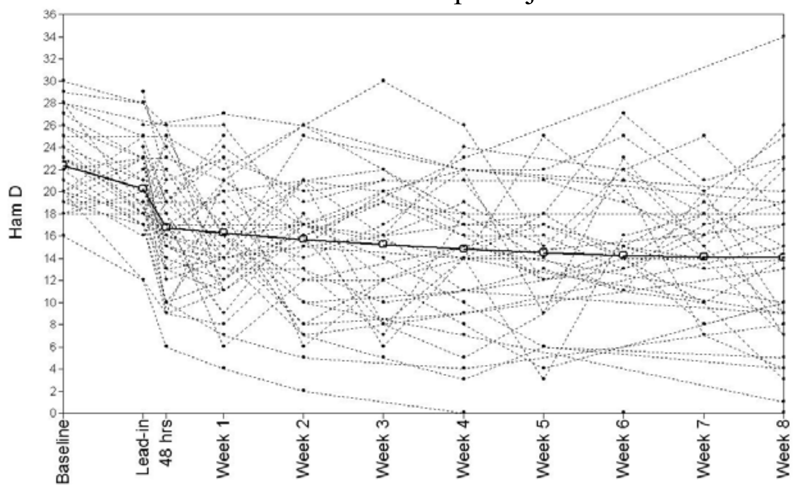


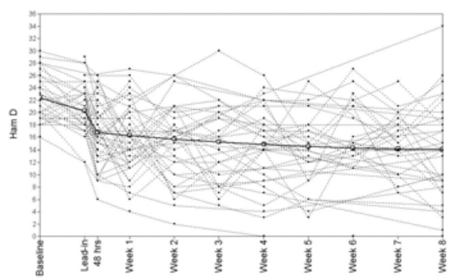
Figure 2. Longitudinal PSA curves estimated from the linear mixed-effects model for the group average (thick solid line) and for each individual in the study (thin solid lines)

Source: Pearson, Morrell, Landis and Carter (1994), Statistics in Medicine

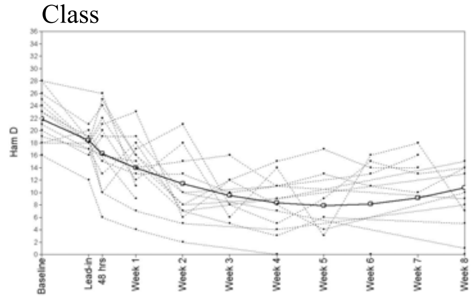
A Clinical Trial Of Antidepressants
Growth Mixture Modeling With Placebo Response
Muthen et al (2008)

Placebo Group Subjects

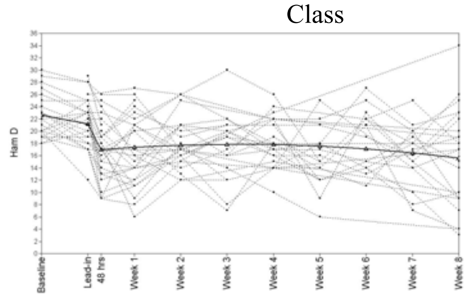




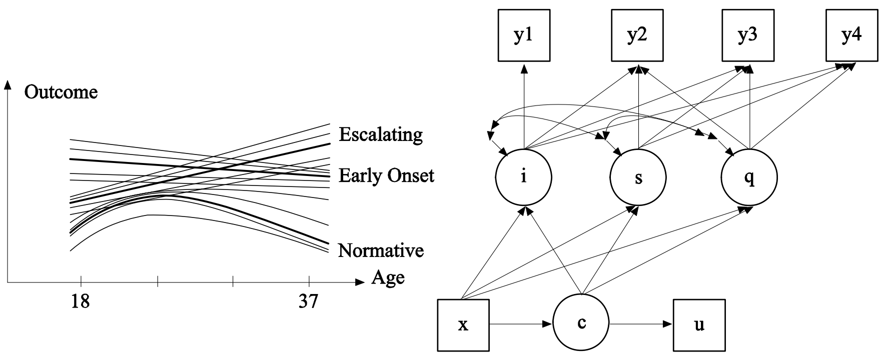
Responder Class



Non-Responder Class



Growth Mixture Modeling Of Developmental Pathways



Growth Mixture Analysis

Generalization of conventional random effect growth modeling (multilevel modeling) to include qualitatively different developments (Muthén & Shedden, 1999 in Biometrics). Combination of conventional growth modeling and cluster analysis (finite mixture analysis).

- Setting
  - Longitudinal data
  - A single or multiple items measured repeatedly
  - Hypothesized trajectory classes (categorical latent variable)
  - Individual trajectory variation within classes

Growth Mixture Analysis (Continued)

- Aim
  - Estimate trajectory shapes
  - Estimate trajectory class probabilities
  - Estimate variation within class
  - Relate class probabilities to covariates
  - Relate within-class variation to covariates
  - Classify individuals into classes (posterior prob's)

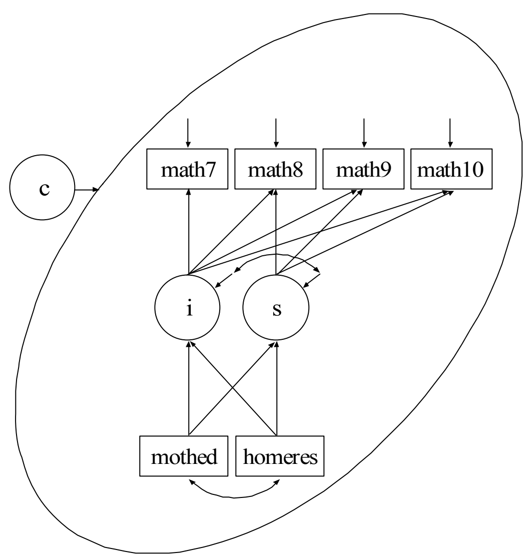
Application: Mathematics achievement, grades 7 – 10 (LSAY) related to mother's education and home resources.
National sample, n = 846.

Strategies For Finding The Number Of Classes In Growth Mixture Modeling

- **Comparing models with different numbers of classes**
  - BIC – low BIC value corresponds to a high loglikelihood value and a parsimonious model
  - TECH11 – Lo-Mendell-Rubin likelihood ratio test (Biometrika, 2001)
  - TECH14 – bootstrapped LRT (Version 4)
- **Residuals and model tests for continuous outcomes**
  - RESIDUAL – class-specific comparisons of model-estimated means, variances, and covariances versus posterior probability-weighted sample statistics
  - TECH12 – class-mixed residuals for univariate skewness and kurtosis
  - TECH13 – multivariate skew and kurtosis model tests

Strategies For Finding The Number Of Classes In Growth Mixture Modeling (Continued)

- **Interpretability and usefulness of the latent classes**
  - Trajectory shapes
  - Number of individuals in each class
  - Number of estimated parameters
  - Substantive theory
  - Auxiliary (external) variables – predictive validity
- **Classification quality**
  - Posterior probabilities – classification table and entropy
  - Individual trajectory classification using pseudo classes (Bandeen-Roche et al., 1997; Muthén et al. in Biostatistics, 2002)

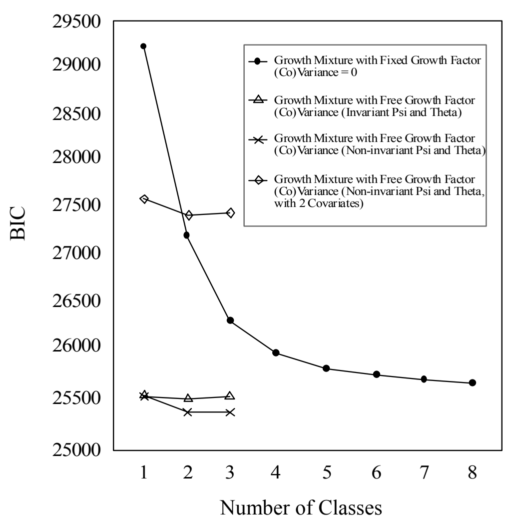


Deciding On The Number Of Classes For The LSAY Growth Mixture Model

n = 935

Number of Classes	1	2	3
Loglikelihood	-11,997.653	-11,864.826	-11,856.220
# parameters	15	29	36
BIC	24,098	23,928	23,959
AIC	24,025	23,788	23,784
Entropy	NA	.468	.474
TECH11 LRT p-value for k-1 classes	NA	.0000	.4041
Multivariate skew p-value	.00	.34	.26
Multivariate kurtosis p-value	.00	.10	.05

Model Fit by BIC: LSAY



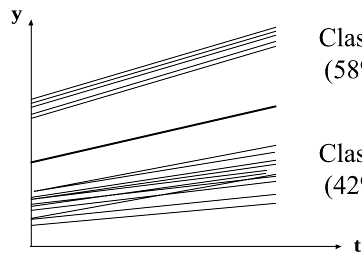
LSAY: Estimated Two-Class Growth Mixture Model

Class 1

Intercept	ON Mothed*	Homeres
Slope	ON Mothed	Homeres*

Class 2

Intercept	ON Mothed*	Homeres*
Slope	ON Mothed	Homeres



Class 2
(58%)

Class 1
(42%)

Conventional Single-Class Analysis

Intercept	ON Mothed*	Homeres*
Slope	ON Mothed	Homeres*

Input For LSAY Two-Class Growth Mixture Model

```
TITLE:      2-class varying slopes on mothed and homeres
            varying Psi varying Theta

DATA:       FILE IS lsay.dat;
            FORMAT IS 3f8 f8.4 8f8.2 2f8.2;

VARIABLE:   NAMES ARE cohort id school weight math7
            math8 math9 math10 att7 att8 att9 att10 gender mothed
            homeres;
            USEOBS = (gender EQ 1 AND cohort EQ 2);
            MISSING = ALL (999);
            USEVAR = math7-math10 mothed homeres;
            CLASSES = c(2);

ANALYSIS:   TYPE = MIXTURE;
```

Input For LSAY Two-Class Growth Mixture Model (Continued)

```
MODEL:       %OVERALL%
             intercpt slope | math7@0 math8@1 math9 math10;
             intercpt slope ON mothed homeres;
             %c#2%
             intercpt slope ON mothed homeres;
             math7-math10 intercpt slope;
             slope WITH intercpt;

OUTPUT:      TECH8 TECH12 TECH13 RESIDUAL;
```


Output Excerpts LSAY Two-Class Growth Mixture Model

Tests Of Model Fit

```
Loglikelihood
             H0 Value                           -11864.825
Information Criteria
             Number of Free Parameters                  29
             Akaike (AIC)                        23787.649
             Bayesian (BIC)                      23928.025
             Sample-Size Adjusted BIC            23835.923
                (n* = (n + 2) / 24)
             Entropy                                 0.468
```

Output Excerpts LSAY Two-Class Growth Mixture Model (Continued)

Classification Information

FINAL CLASS COUNTS AND PROPORTIONS OF TOTAL SAMPLE SIZE

Class 1	392.19327	0.41946
Class 2	542.80673	0.58054

CLASSIFICATION OF INDIVIDUALS BASED ON THEIR MOST LIKELY CLASS MEMBERSHIP

Class Counts and Proportions

Class 1	342	0.36578
Class 2	593	0.63422

Average Latent Class Probabilities for Most Likely Latent Class Membership (Row) by Latent Class (Column)

	1	2
Class 1	0.853	0.147
Class 2	0.170	0.830

Output Excerpts LSAY Two-Class Growth Mixture Model (Continued)

TECHNICAL 12 OUTPUT

ESTIMATED MIXED MODEL AND RESIDUALS (OBSERVED – EXPECTED)

Observed Skewness

MATH7	MATH8	MATH9	MATH10	MOTHED	HOMERES
-0.184	-0.312	-0.349	-0.471	1.214	-0.087

Estimated Mixed Skewness

MATH7	MATH8	MATH9	MATH10	MOTHED	HOMERES
-0.160	-0.190	-0.348	-0.517	0.002	-0.012

Output Excerpts LSAY Two-Class Growth Mixture Model (Continued)

```
TECHNICAL 13 OUTPUT

   SKEW AND KURTOSIS TESTS OF MODEL FIT

   TWO-SIDED MULTIVARIATE SKEW TEST OF FIT
         Sample Value                        1.245
         Mean                                0.999
         Standard Deviation                  0.275
         P-Value                            0.3400
   TWO-SIDED MULTIVARIATE KURTOSIS TEST OF FIT
         Sample Value                       29.612
         Mean                               27.842
         Standard Deviation                  1.015
         P-Value                            0.1000
```


Output Excerpts LSAY Two-Class Growth Mixture Model (Continued)

Model Results

```
                         Estimates          S.E.      Est./S.E.
Class 1

 INTERCPT |
    MATH7                    1.000          .000           .000
    MATH8                    1.000          .000           .000
    MATH9                    1.000          .000           .000
    MATH10                   1.000          .000           .000

 SLOPE    |
    MATH7                     .000          .000           .000
    MATH8                    1.000          .000           .000
    MATH9                    2.422          .133         18.157
    MATH10                   3.580          .204         17.570

 INTERCPT ON
    MOTHED                   1.656          .626          2.645
    HOMERES                   .720          .377          1.911

 SLOPE    ON
    MOTHED                    .146          .154           .953
    HOMERES                   .228          .087          2.626
```

Output Excerpts LSAY Two-Class Growth Mixture Model (Continued)

	Estimates	S.E.	Est./S.E.
SLOPE WITH			
INTERCPT	.727	1.643	.443
Residual Variances			
MATH7	17.198	2.840	6.055
MATH8	15.257	2.077	7.347
MATH9	24.170	3.294	7.337
MATH10	49.112	10.037	4.893
INTERCPT	54.297	6.094	8.910
SLOPE	1.643	.627	2.620
Intercepts			
MATH7	.000	.000	.000
MATH8	.000	.000	.000
MATH9	.000	.000	.000
MATH10	.000	.000	.000
INTERCPT	42.733	1.648	25.922
SLOPE	.816	.366	2.228

Output Excerpts LSAY Two-Class Growth Mixture Model (Continued)

	Estimates	S.E.	Est./S.E.
Class 2			
INTERCPT |			
MATH7	1.000	.000	.000
MATH8	1.000	.000	.000
MATH9	1.000	.000	.000
MATH10	1.000	.000	.000
SLOPE |			
MATH7	.000	.000	.000
MATH8	1.000	.000	.000
MATH9	2.422	.133	18.157
MATH10	3.580	.204	17.570
INTERCPT ON			
MOTHED	2.085	.376	5.545
HOMERES	1.805	.285	6.334
SLOPE ON			
MOTHED	.028	.079	.358
HOMERES	.054	.058	.943

Output Excerpts LSAY Two-Class Growth Mixture Model (Continued)

	Estimates	S.E.	Est./S.E.
SLOPE WITH			
INTERCPT	.478	.575	.831
Residual Variances			
MATH7	12.248	1.587	7.720
MATH8	11.375	1.244	9.141
MATH9	8.014	1.420	5.642
MATH10	7.349	1.312	5.600
INTERCPT	32.431	4.651	6.973
SLOPE	.191	.264	.724
Intercepts			
MATH7	.000	.000	.000
MATH8	.000	.000	.000
MATH9	.000	.000	.000
MATH10	.000	.000	.000
INTERCPT	45.365	1.461	31.054
SLOPE	2.847	.316	9.015

LATENT CLASS REGRESSION MODEL PART

Means	Estimates	S.E.	Est./S.E.
C#1	-.325	.306	-1.063

Output Excerpts LSAY Two-Class Growth Mixture Model (Continued)

Residuals

ESTIMATED MODEL AND RESIDUALS (OBSERVED – ESTIMATED) FOR CLASS 1

Model Estimated Means

MATH7	MATH8	MATH9	MATH10	MOTHED	HOMERES
48.650	50.486	53.095	55.221	2.255	3.030

Residuals for Means

MATH7	MATH8	MATH9	MATH10	MOTHED	HOMERES
-0.113	0.072	0.236	-0.389	0.000	0.000

Model Estimated Covariances

	MATH7	MATH8	MATH9	MATH10	MOTHED
MATH7	76.058				
MATH8	60.370	78.945			
MATH9	62.515	68.408	100.958		
MATH10	64.263	72.253	83.615	141.979	
MOTHED	1.751	1.962	2.263	2.507	0.907
HOMERES	2.309	2.908	3.760	4.454	0.345

Output Excerpts LSAY Two-Class Growth Mixture Model (Continued)

	HOMERES
HOMERES	2.412

Residuals for Covariances

	MATH7	MATH8	MATH9	MATH10	MOTHED
MATH7	-0.153				
MATH8	-0.109	-0.143			
MATH9	0.413	0.572	0.451		
MATH10	-0.701	-0.614	-0.254	-0.338	
MOTHED	0.210	-0.252	-0.021	0.256	0.000
HOMERES	0.289	-0.254	-0.358	0.720	0.000

	HOMERES
HOMERES	0.000

Further Readings On Growth Mixture Modeling

Muthén, B. (2001). Second-generation structural equation modeling with a combination of categorical and continuous latent variables: New opportunities for latent class/latent growth modeling. In Collins, L.M. & Sayer, A. (Eds.), New methods for the analysis of change (pp. 291-322). Washington, D.C.: APA. (#82)

Muthén, B. (2001). Latent variable mixture modeling. In G. A. Marcoulides & R. E. Schumacker (eds.), New developments and techniques in structural equation modeling (pp. 1-33). Lawrence Erlbaum Associates. (#86)

Muthén, B. (2002). Beyond SEM: General latent variable modeling. Behaviormetrika, 29, 81-117. (#96)

Muthén, B. (2004). Latent variable analysis: Growth mixture modeling and related techniques for longitudinal data. In D. Kaplan (ed.), Handbook of quantitative methodology for the social sciences (pp. 345-368). Newbury Park, CA: Sage Publications. (#100)

Further Readings On Growth Mixture Modeling (Continued)

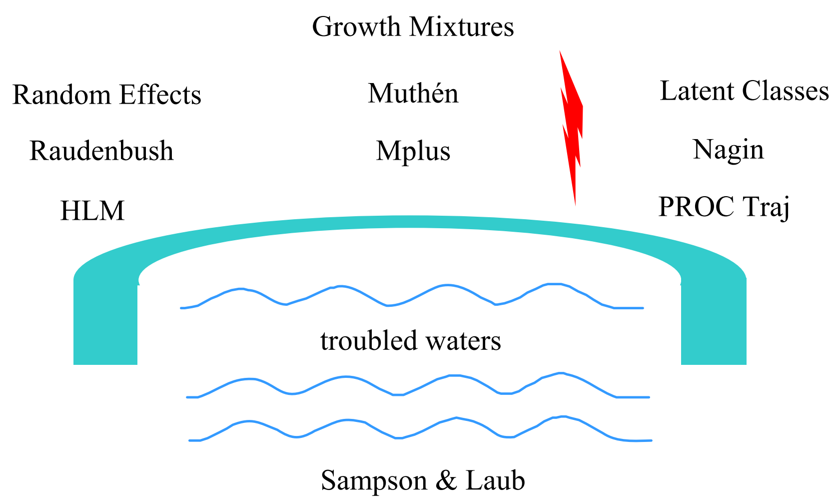
Muthén, B. & Asparouhov, T. (2008). Growth mixture modeling: Analysis with non-Gaussian random effects. In Fitzmaurice, G., Davidian, M., Verbeke, G. & Molenberghs, G. (eds.), Longitudinal Data Analysis, pp. 143-165. Boca Raton: Chapman & Hall/CRC Press.

Muthén, B. & Muthén, L. (2000). Integrating person-centered and variable-centered analysis: growth mixture modeling with latent trajectory classes. Alcoholism: Clinical and Experimental Research, 24, 882-891. (#85)

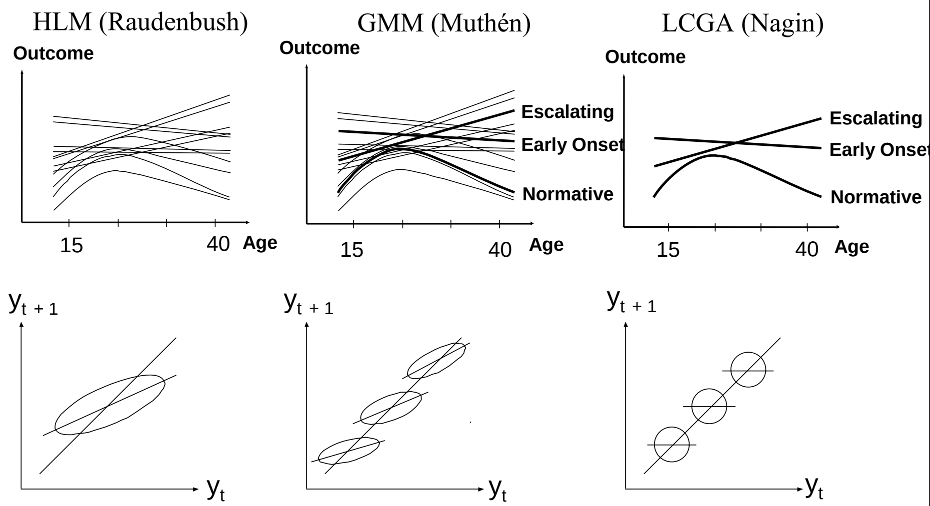
Muthén, B. & Shedden, K. (1999). Finite mixture modeling with mixture outcomes using the EM algorithm. Biometrics, 55, 463-469. (#78)

LCGA Vs GMM Modeling Without Vs With Random Effects

Growth Modeling Paradigms: Debate In Criminology And Annals AAPSS



Growth Modeling Paradigms (Continued)



- Replace rhetoric with statistics – let likelihood decide
- HLM and LCGA are special cases of GMM

Philadelphia Crime Data
ZIP Growth Mixture Modeling

- 13,160 males ages 4 - 26 born in 1958 (Moffitt, 1993; Nagin & Land, 1993)
- Annual counts of police contacts
- Individuals with more than 10 counts in any given year deleted (n=13,126)
- Data combined into two-year intervals

Zero-Inflated Poisson (ZIP)
Growth Mixture Modeling Of Counts

$$u_{ti} = \begin{cases} 0 & \text{with probability} \quad \pi_{ti} \\ \text{Poisson } (\lambda_{ti}) & \text{with probability} \quad 1 - \pi_{ti} \end{cases}$$

$$\ln \lambda_{ti \mid C_i = c} = \eta_{0i} + \eta_{1i} a_{ti} + \eta_{2i} a_{ti}^2$$

$$\eta_{0i \mid C_i = c} = \alpha_{0c} + \zeta_{0i}$$

$$\eta_{1i \mid C_i = c} = \alpha_{1c} + \zeta_{1i}$$

$$\eta_{2i \mid C_i = c} = \alpha_{2c} + \zeta_{2i}$$

In Mplus, $\pi_{ti} = P(u\#_{ti} = 1)$, where *u#* is a binary latent inflation variable and *u#=1* indicates that the individual is unable to assume any value except 0.

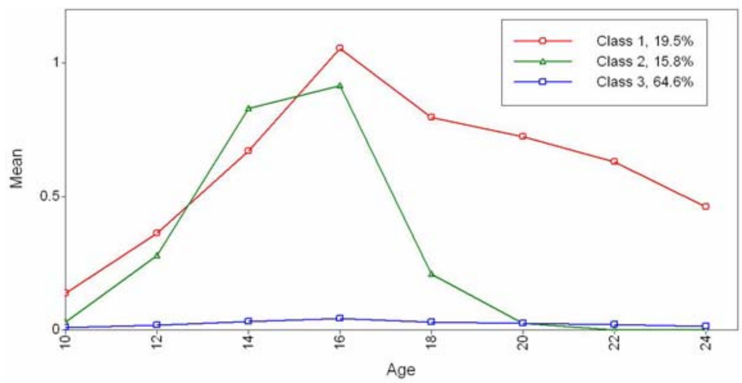
ZIP Growth Factor Scale

- The ZIP growth model is expressed in terms of log rate: $\ln \lambda$
- For a Poisson variable, the mean count = $\lambda = \exp(\ln \lambda)$
  - For example, at the centering point $\ln \lambda$ = [i], so that the mean count = exp([i]). A large negative value of [i] gives a mean count close to zero
- For a ZIP variable, the mean count = $\lambda - \lambda * \pi$ where π = P(being in the zero class)

ZIP Modeling Of Philadelphia Crime: Log-Likelihood And BIC Comparisons For GMM And LCGA

Model	Log-Likelihood	# Parameters	BIC	# Significant Residuals
1-class GMM	-40,606	17	81,373	5
2-class GMM	-40,422	21	81,044	4
3-class GMM	-40,283	25	80,803	1
4-class GMM	-40,237	29	80,748	0
4-class LCGA	-40,643	23	81,503	4
5-class LCGA	-40,483	27	81,222	3
6-class LCGA	-40,410	31	81,114	3
7-class LCGA	-40,335	35	81,003	2
8-class LCGA	-40,263	39	80,896	1

Three-Class ZIP GMM for Philadelphia Crime



Input Excerpts Three-Class ZIP GMM For Philadelphia Crime

```
               USEVAR = y10 y12 y14 y16 y18 y20 y22 y24;
               !y10 = ages 10-11, y12 = ages y12-13, etc
               IDVAR = cohortid;
               USEOBS = y10 LE 10 AND y12 LE 10 AND y14 LE 10
               AND y16 LE 10 AND y18 LE 10 AND y20 LE 10
               AND y22 LE 10 AND y24 LE 10;
               COUNT = y10-y24(i);
               CLASSES = c(3);

ANALYSIS:      TYPE = MIXTURE;
               ALGORITHM = INTEGRATION;
               PROCESS = 4(STARTS);
               INTEGRATION = 10;
               STARTS = 50 5;
               INTERACTIVE = control.dat;
```

Input Excerpts Three-Class ZIP GMM For Philadelphia Crime

```
MODEL:        %OVERALL%

              i s q | y10@0 y12@.1 y14@.2 y16@.3 y18@.4
              y20@.5 y22@.6 y24@.7;

OUTPUT:       TECH1 TECH10;

PLOT:         TYPE = PLOT3;
              SERIES = y10-y24(s);
```


Output Excerpts Three-Class ZIP GMM For Philadelphia Crime

	Estimates	S.E.	Est./S.E.
Latent Class 1			
Means			
I	-2.776	0.157	-17.666
S	12.774	0.658	19.419
Q	-15.081	0.752	-20.049
Y10#1	0.444	0.138	3.208
Y12#1	-0.853	0.145	-5.877
Y14#1	-1.637	0.143	-11.456
Y16#1	-2.84	0.284	-9.994
Y18#1	-0.536	0.079	-6.798
Y20#1	-0.644	0.088	-7.361
Y22#1	-1.535	0.157	-9.771
Y24#1	-15.000	0.000	0.000
Variances			
I	3.472	0.315	11.007
S	43.64	5.167	8.446
Q	53.768	7.258	7.408

Output Excerpts Three-Class ZIP GMM For Philadelphia Crime

	Estimates	S.E.	Est./S.E.
Latent Class 2			
Means			
I	-4.342	0.327	-13.265
S	30.003	2.231	13.448
Q	-56.706	4.073	-13.92
Y10#1	0.444	0.138	3.208
Y12#1	-0.853	0.145	-5.877
Y14#1	-1.637	0.143	-11.456
Y16#1	-2.84	0.284	-9.994
Y18#1	-0.536	0.079	-6.798
Y20#1	-0.644	0.088	-7.361
Y22#1	-1.535	0.157	-9.771
Y24#1	-15	0	0
Variances			
I	3.472	0.315	11.007
S	43.64	5.167	8.446
Q	53.768	7.258	7.408

Output Excerpts Three-Class ZIP GMM For Philadelphia Crime

	Estimates	S.E.	Est./S.E.
Latent Class 3			
Means			
I	-5.607	0.202	-27.821
S	11.362	0.938	12.107
Q	-14.387	1.175	-12.249
Y10#1	0.444	0.138	3.208
Y12#1	-0.853	0.145	-5.877
Y14#1	-1.637	0.143	-11.456
Y16#1	-2.84	0.284	-9.994
Y18#1	-0.536	0.079	-6.798
Y20#1	-0.644	0.088	-7.361
Y22#1	-1.535	0.157	-9.771
Y24#1	-15	0	0
Variances			
I	3.472	0.315	11.007
S	43.64	5.167	8.446
Q	53.768	7.258	7.408

Further Readings On LCGA Vs GMM

Kreuter, F. & Muthen, B. (2008). Analyzing criminal trajectory profiles: Bridging multilevel and group-based approaches using growth mixture modeling. Journal of Quantitative Criminology, 24, 1-31.

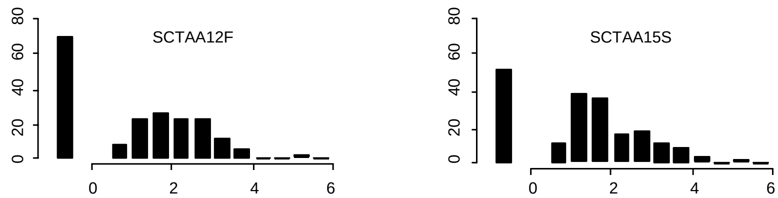
Kreuter, F. & Muthen, B. (2008). Longitudinal modeling of population heterogeneity: Methodological challenges to the analysis of empirically derived criminal trajectory profiles. In Hancock, G. R., & Samuelsen, K. M. (Eds.), Advances in latent variable mixture models, pp. 53-75. Charlotte, NC: Information Age Publishing, Inc.

Muthén, B. & Asparouhov, T. (2008). Growth mixture modeling: Analysis with non-Gaussian random effects. In Fitzmaurice, G., Davidian, M., Verbeke, G. & Molenberghs, G. (eds.), Longitudinal Data Analysis, pp. 143-165. Boca Raton: Chapman & Hall/CRC Press.

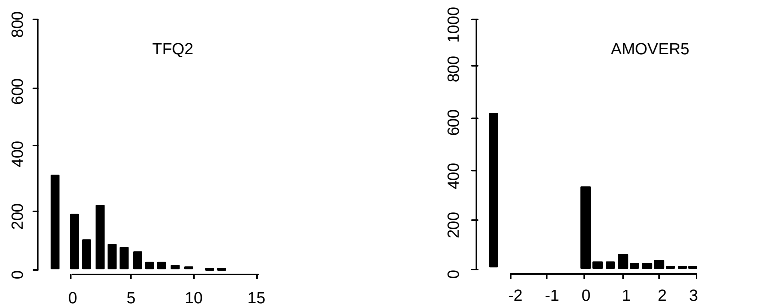
Growth Mixture Modeling Caveats

Two Types Of Distributions

(1) Normal mixture components

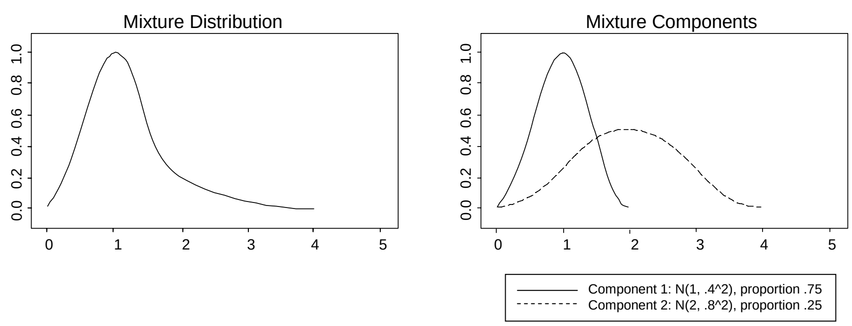


(2) Preponderance of zeroes



Substantive Classes Or Non Normality?

Non-normality for mixture, normality for mixture components



- McLachlan & Peel (2000, pp. 14-17)
  - Blood pressure debate - is hypertension a disease?
- Bauer & Curran (2003)
  - Latent trajectory classes due to non-normality in the outcomes

General Growth Mixture Modeling

General Growth Mixture Modeling (GGMM)

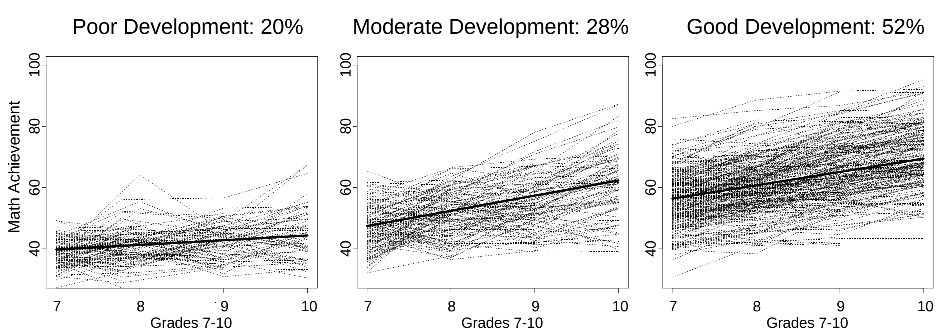
GGMM goes beyond conventional random effect growth modeling by using latent trajectory classes which

- Allow for heterogeneity with respect to
  - Growth functions – different classes correspond to different growth shapes
  - Antecedents – different background variables have different importance for different classes
  - **Consequences – class membership predicts later outcomes**
- Allow for prediction of trajectory class membership
- Allow for confirmatory clustering
  - With respect to parameters – describing curve shapes
  - With respect to typical individuals – known classes
- Allow for classification of individuals
  - Early prediction of problematic development
- Allow for enhanced preventive intervention analysis
  - Different classes benefit differently and can receive different treatments

General Growth Mixture Modeling (Continued)

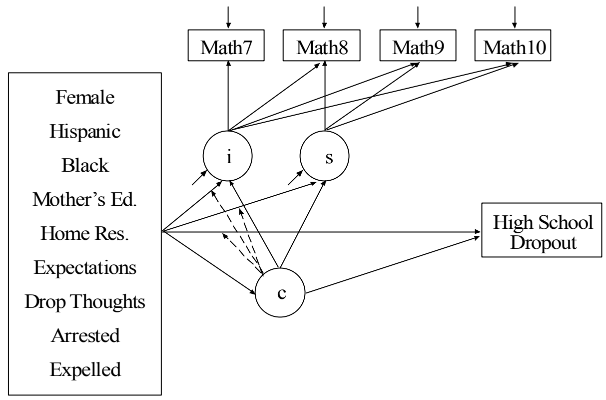
- Applications
  - LSAY math achievement development and high school dropout
  - The development of heavy drinking ages 18-30 (NLSY) related to antecedents and consequences. National sample, n = 922

Mplus Graphics For LSAY Math Achievement Trajectory Classes

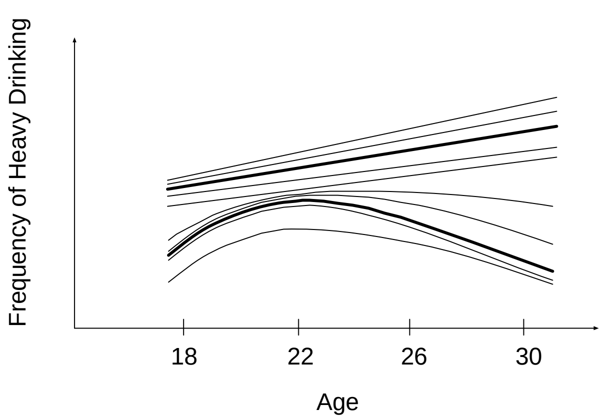


Dropout:	69%	8%	1%

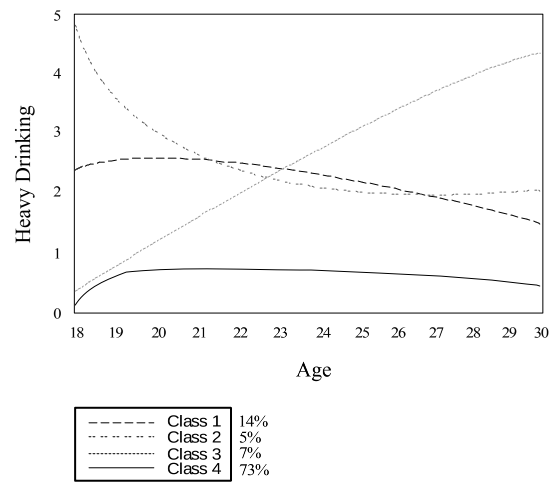
LSAY Math Achievement Trajectory Classes



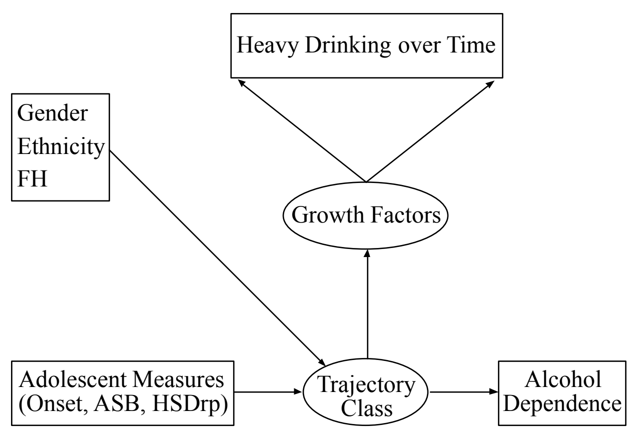
Example: NLSY Heavy Drinking Two Latent Trajectory Classes



NLSY: Heavy Drinking, Cohort 64



NLSY: Antecedents And Consequences



Multinomial Logistic Regression Of c ON x

The multinomial logistic regression model expresses the probability that individual *i* falls in class *k* of the latent class variable *c* as a function of the covariate *x*,

$$P(c_j = k \mid x_i) = \frac{e^{\alpha_k + \gamma_k x_i}}{\sum_{s=1}^{K} e^{\alpha_s + \gamma_s x_i}}\ , \qquad (90)$$

where $\alpha_K = 0$, $\gamma_K = 0$ so that $e^{\alpha_K + \gamma_K x_i} = 1$.

This implies that the log odds comparing class *k* to the last class *K* is

$$log[P(c_i = k \mid x_i)/P(c_i = K \mid x_i)] = \alpha_k + \gamma_k x_i\ . \qquad (91)$$

Heavy Drinking And Alcohol Dependence NLSY Cohort64 (n=922)

HD Class on Covariates						
	HD Classes					
	1 (Down)		2 (High18)		3(Up)	
	Est.	z	Est.	z	Est.	z
Male	**1.21**	5.52	**1.25**	3.48	**1.45**	4.73
Black	**-0.89**	-3.43	**-3.14**	-2.86	-0.06	-0.17
Hisp	**-0.65**	-2.22	-0.35	-0.86	-0.01	-0.03
ES	**1.24**	4.79	**2.05**	5.72	0.71	1.78
FH1	0.03	0.09	-0.21	-0.41	-0.08	-0.16
FH23	0.04	0.15	0.25	0.56	0.08	0.23
FH123	-0.23	-0.58	**1.18**	2.59	**1.00**	2.60
HSDRP	0.57	1.98	0.32	0.76	**0.91**	2.93
Coll	-0.07	-0.31	**-1.31**	-2.85	**-1.08**	-2.59

Heavy Drinking And Alcohol Dependence NLSY Cohort64 (n=922) (Continued)

Alcohol Dependence as a Function of Heavy Drinking Class

	Threshold	Probability	Odds	Odds Ratio
HD Class 1 (Down)	1.631	0.164	0.196	3.92
HD Class 2 (High 18)	1.041	0.261	0.353	7.06
HD Class 3 (Up)	-0.406	0.600	1.500	30.00
HD Class 4 (Norm)	2.987	0.048	0.050	1.00

Probability = $1/(1 + e^{-Logit})$ where Logit = − threshold

Odds = Probability/(1 - Probability)

Odds Ratio = Odds (class k)/Odds (class K) for K = 4

Input Excerpts NLSY Growth Mixture Model With Covariates And A Distal Outcome

```
TITLE:      NLSY for cohort 64 quadratic growth mixture model with
            covariates centered at 25: four-class model of heavy
            drinking with classes predicting dep94

VARIABLE:   CLASSES = c(4);
            CATEGORICAL IS dep94;  ! dep94 is alcohol dependence

ANALYSIS:   TYPE = MIXTURE;

MODEL:      %OVERALL%
            c#1-c#3 ON male black hisp es fh1 fh23 fh123 hsdrp coll;
            i s1 s2 | hd82@-3.008 hd83@-2.197 hd84@-1.621 hd88@-.235
                      hd89@.000 hd94@.884;
```

Input Excerpts NLSY Growth Mixture Model With Covariates And A Distal Outcome (Continued)

```
!       log age scale: x_t = a*(ln(t-b) – ln(c-b));
!       where t is time, a and b are constants to fit the mean curve
!       (chosen as a = 2 and b = 16), and c is the centering age,
!       here set at 25.
        %c#1%                              ! Not needed
        [dep94$1*1 i*2 s1*-.5 s2*-.1];     ! Not needed
        %c#2%                              ! Not needed
        [dep94$1*0 i*1 s1*-.2 s2*-.3];     ! Not needed
        %c#3%                              ! Not needed
        [dep94$1*.6 i*3 s1*1.5 s2*.2];     ! Not needed
        %c#4%                              ! Not needed
        [dep94$1*2 i*.6 s1*-.2 s2*-.1];    ! Not needed
```


Output Excerpts NLSY Growth Mixture Model With Covariates And A Distal Outcome

	Est.	S.E.	Est./S.E.
C#1 ON			
MALE	1.214	.220	5.515
BLACK	-.886	.258	-3.434
HISP	-.645	.290	-2.223
ES	1.240	.259	4.789
FH1	.026	.291	.088
FH23	.039	.261	.149
FH123	-.233	.399	-.583
HSDRP	.566	.286	1.976
COLL	-.071	.231	-.308

Output Excerpts NLSY Growth Mixture Model With Covariates And A Distal Outcome (Continued)

C#2 ON			
MALE	1.248	.359	3.481
BLACK	-3.138	1.097	-2.860
HISP	-.346	.401	-.864
ES	2.045	.357	5.722
FH1	-.211	.514	-.410
FH23	.247	.444	.556
FH123	1.178	.456	2.585
HSDRP	.323	.428	.756
COLL	-1.311	.460	-2.851
C#3 ON			
MALE	1.454	.308	4.727
BLACK	-.059	.344	-.171
HISP	-.011	.369	-.030
ES	.712	.399	1.784
FH1	-.079	.502	-.157
FH23	.084	.364	.232
FH123	1.004	.387	2.596
HSDRP	.913	.312	2.926
COLL	-1.075	.414	-2.594

Output Excerpts NLSY Growth Mixture Model With Covariates And A Distal Outcome (Continued)

	Est.	S.E.	Est./S.E.
Class 1			
Thresholds			
DEP94$1	1.631	0.248	6.574
Class 2			
Thresholds			
DEP94$1	1.041	0.338	3.077
Class 3			
Thresholds			
DEP94$1	-0.406	0.272	-1.493
Class 4			
Thresholds			
DEP94$1	2.987	0.208	14.392

Output Excerpts NLSY Growth Mixture Model With Covariates And A Distal Outcome (Continued)

Classification Information

FINAL CLASS COUNTS AND PROPORTIONS OF TOTAL SAMPLE SIZE

Class 1	135.95653	0.14746
Class 2	45.86689	0.04975
Class 3	74.68767	0.08101
Class 4	665.48891	0.72179

CLASSIFICATION OF INDIVIDUALS BASED ON THEIR MOST LIKELY CLASS MEMBERSHIP

Class Counts and Proportions

Class 1	134	0.14534
Class 2	46	0.04989
Class 3	72	0.07809
Class 4	670	0.72668

Average Latent Class Probabilities for Most Likely Latent Class Membership (Row) by Latent Class (Column)

	1	2	3	4
Class 1	0.994	0.000	0.000	0.005
Class 2	0.003	0.997	0.000	0.000
Class 3	0.007	0.000	0.947	0.047
Class 4	0.003	0.000	0.010	0.987

General Growth Mixture Modeling With Sequential Processes

- New setting:
  - Sequential, linked processes
- New aims:
  - Using an earlier process to predict a later process
  - Early prediction of failing class

Application: General growth mixture modeling of first- and second-grade reading skills and their Kindergarten precursors; prediction of reading failure (Muthén, Khoo, Francis, Boscardin, 1999). Suburban sample, n = 410.

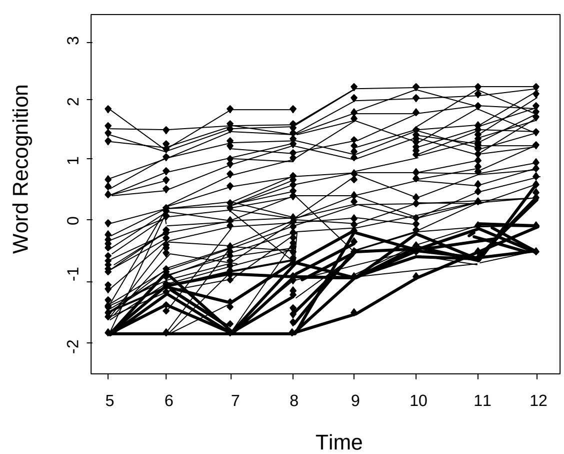
Assessment Of Reading Skills Development

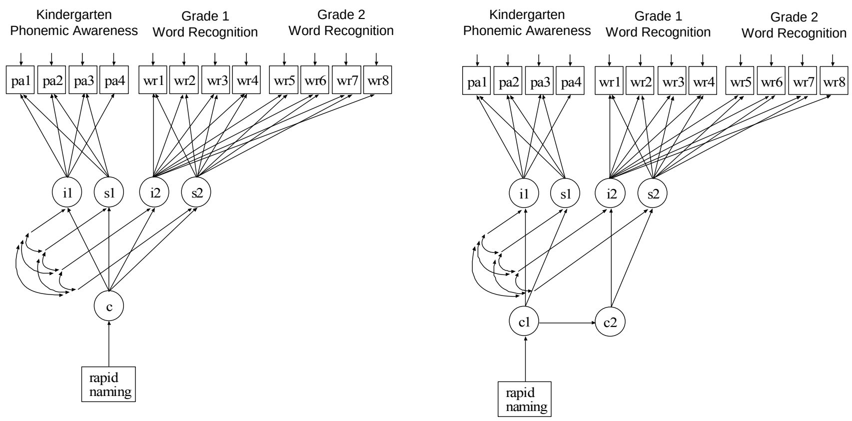
- Longitudinal multiple-cohort design involving approximately 1000 children with measurements taken four times a year from Kindergarten through grade two (October, December, February, April)
- Grade 1 – Grade 2: reading and spelling skills
- Precursor skills: phonemic awareness (Kindergarten, Grade 1, Grade 2), letters/names/sounds (Kindergarten only), rapid naming
- Standardized reading comprehension tests at the end of Grade 1 and Grade 2 (May).

Three research hypotheses (EARS study; Francis, 1996):

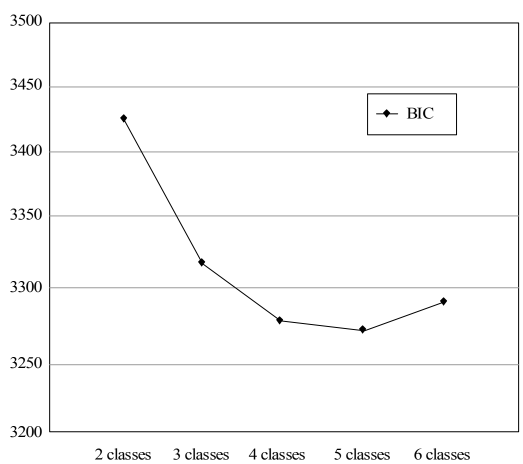
- Kindergarten children will differ in their growth and development in precursor skills
- The rate of development of the precursor skills will relate to the rate of development and the level of attainment of reading and spelling skills – and the individual growth rates in reading and spelling skills will predict performance on standardized tests of reading and spelling
- The use of growth rates for skills and precursors will allow for earlier identification of children at risk for poor academic outcomes and lead to more stable predictions regarding future academic performance

Word Recognition Development In Grades 1 And 2





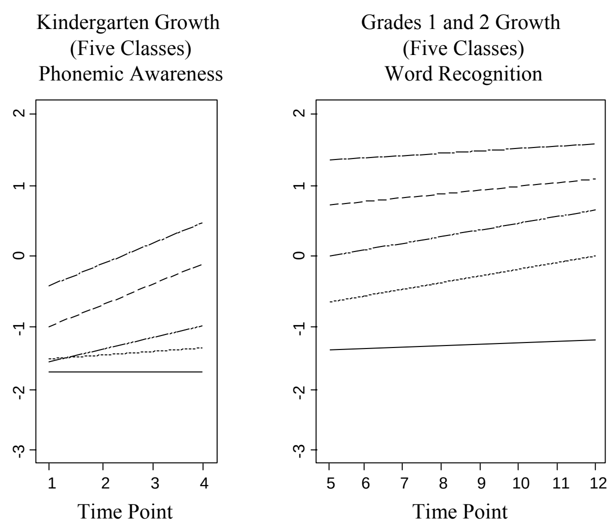
BIC Curve For Reading Skills Development



Input For Growth Mixture Model For Reading Skills Development

```
TITLE:     Growth mixture model for reading skills development

DATA:      FILE IS newran.dat;

VARIABLE:  NAMES ARE gender eth wc pa1-pa4 wr1-wr8 l1-l4 s1 r1 s2 r2
           rnaming1 rnaming2 rnaming3 rnaming4;
           USEVAR = pa1-wr8 rnaming4;
           MISSING ARE ALL (999);
           CLASSES = c(5);

ANALYSIS:  TYPE = MIXTURE;

MODEL:     %OVERALL%
           i1 s1 | pa1@-3 pa2@-2 pa3@-1 pa4@0;
           i2 s2 | wr1@-7 wr2@-6 wr3@-5 wr4@-4 wr5@-3 wr6@-2
                   wr7@-1 wr8@0;
           c#1-c#4 ON rnaming4;

OUTPUT:    TECH8;
```


Five Classes Of Reading Skills Development



How Early Can A Good Classification Be Made?

Focus on Class 1, the failing class.

1. Estimate full growth mixture model for Kindergarten, Grade 1, and Grade 2 outcomes
2. Use the estimated full model to classify students into classes based on the posterior probabilities for each class, where a student is classified into the class with the largest posterior probability.
3. Classify students using early information by holding parameters fixed at the estimates from the full model of Step 1 and classifying individuals using Kindergarten information only, adding Grade 1 outcomes, adding Grade 2 outcomes
4. Study quality of early classification by cross-tabulating individuals classified as in Steps 2 and 3 (sensitivity and specificity)

Sensitivity And Specificity Of Early Classification

		Full Model					Total
		1.00	2.00	3.00	4.00	5.00	
K Only	1.00	28	7	3			38
	2.00	10	29	16			55
	3.00	8	33	100	25		166
	4.00		1	24	63	17	105
	5.00		1	1	12	32	46
Total		46	71	144	100	49	410

		Full Model					Total
		1.00	2.00	3.00	4.00	5.00	
K + 1 Only	1.00	28	7	3			38
	2.00	15	44	24			83
	3.00	3	20	112	20		155
	4.00			5	79	4	88
	5.00				1	45	46
Total		46	71	144	100	49	410

		Full Model					Total
		1.00	2.00	3.00	4.00	5.00	
K + 2 Only	1.00	28	8				36
	2.00	16	54	22			92
	3.00	2	9	119	7		137
	4.00			4	91	4	99
	5.00				2	45	47
Total		46	71	145	100	49	411

Sensitivity And Specificity Of Early Classification (Continued)

		Full Model 1.00	2.00	3.00	4.00	5.00	Total
K + 3	1.00	37	12				49
Only	2.00	9	53	8			70
	3.00		6	136	4		146
	4.00			1	95	1	97
	5.00				1	48	49
Total		46	71	145	100	49	411

		Full Model 1.00	2.00	3.00	4.00	5.00	Total
K + 4	1.00	45	11				56
Only	2.00	1	57	3			61
	3.00		3	141	2		146
	4.00			1	97		98
	5.00				1	49	50
Total		46	71	145	100	49	411

		Full Model 1.00	2.00	3.00	4.00	5.00	Total
K + 5	1.00	45	3				48
Only	2.00	1	66				67
	3.00		2	145	1		148
	4.00				98		98
	5.00				1	49	50
Total		46	71	145	100	49	411

Sensitivity And Specificity Of Early Classification (Continued)

		Full Model 1.00	2.00	3.00	4.00	5.00	Total
K + 6	1.00	46					46
Only	2.00		69				69
	3.00		1	145	1		147
	4.00				98		98
	5.00				1	49	50
Total		46	70	145	100	49	410

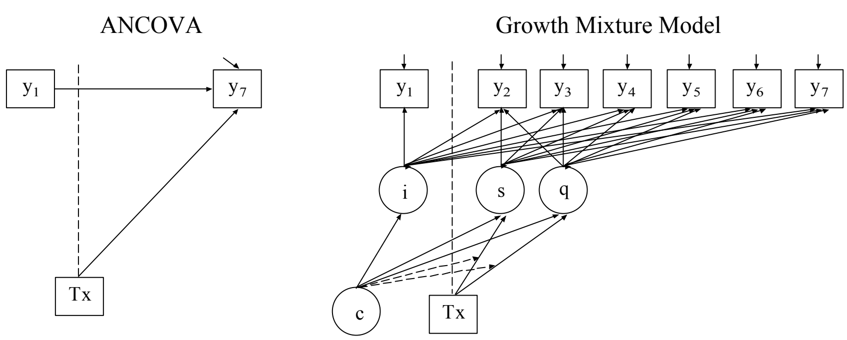
Growth Mixtures In Randomized Trials

Different treatment effects in different trajectory classes

Muthén, B., Brown, C.H., Masyn, K., Jo, B., Khoo, S.T., Yang, C.C., Wang, C.P. Kellam, S., Carlin, J., & Liao, J. (2002). General growth mixture modeling for randomized preventive interventions. Biostatistics, 3, 459-475.

See also Muthen & Curran, 1997 for monotonic treatment effects

Modeling Treatment Effects



- GMM: treatment changes trajectory shape

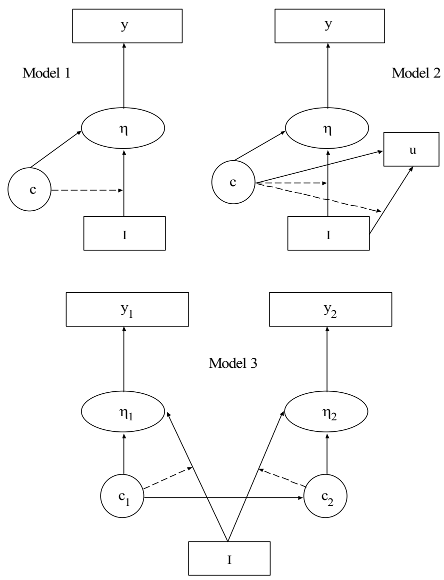


Figure 1. *Path Diagrams for Models 1 - 3*

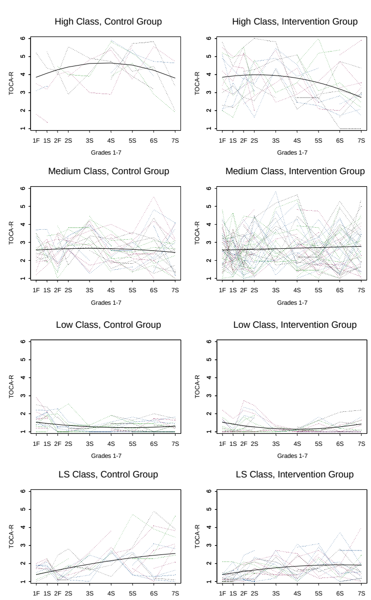
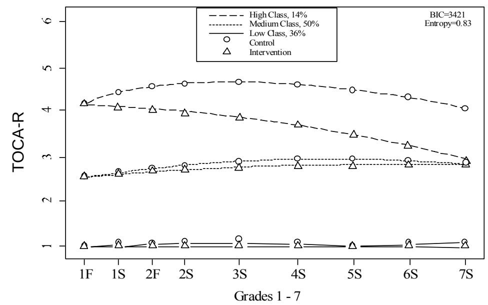
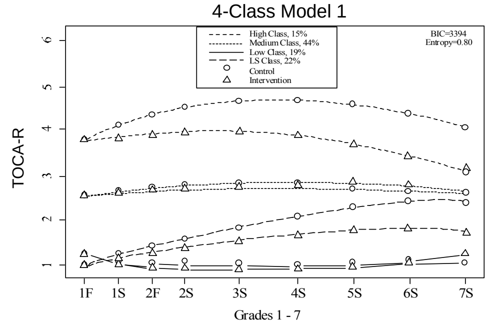


Figure 6. *Estimated Mean Growth Curves and Observed Trajectories for 4-Class model 1 by Class and Intervention Status*

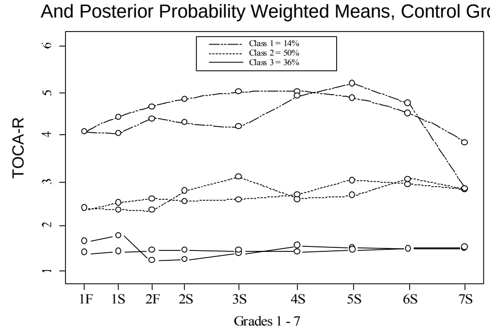
3-Class Model 1



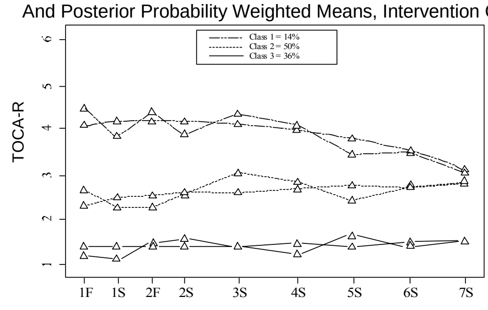
4-Class Model 1



3-Class Model Estimated Mean Class Trajectories And Posterior Probability Weighted Means, Control Group



3-Class Model Estimated Mean Class Trajectories And Posterior Probability Weighted Means, Intervention Group



Input For Growth Mixtures In Randomized Trials

```
TITLE:     growth mixtures in randomized trials
DATA:      FILE IS toca.dat;
VARIABLE:  NAMES ARE sctaa11f sctaa11s sctaa12f sctaa12s sctaa13s
           sctaa14s sctaa15s sctaa16s sctaa17s intngrp;
           MISSING ARE ALL (999);
           USEVARIABLES ARE sctaa11f-sctaa17s tx;
           CLASSES = c(3);
DEFINE:    tx = (intngrp==4);
ANALYSIS:  TYPE = MIXTURE;
MODEL:     %OVERALL%
           ac bc qc | sctaa11f@0 sctaa11s@0.5 sctaa12f@1
           sctaa12s@1.5 sctaa13s@2.5 sctaa14s@3.5 sctaa15s@4.5
           sctaa16s@5.5 sctaa17s@6.5;
           qc@0;
           bc qc ON tx;
           sctaa11f WITH sctaa11s; sctaa12f WITH sctaa12s;
```


Input For Growth Mixtures In Randomized Trials (Continued)

```
           %c#1%
           [ac*3 bc qc]; bc qc ON tx;
           %c#2%
           [ac*2 bc qc]; bc qc ON tx;
           %c#3%
           [ac*1 bc qc]; bc qc ON tx;
           ac sctaa11f-sctaa17s;
```

Further Readings On General Growth Mixture Modeling

Muthén, B. & Asparouhov, T. (2008). Growth mixture modeling: Analysis with non-Gaussian random effects. In Fitzmaurice, G., Davidian, M., Verbeke, G. & Molenberghs, G. (eds.), Longitudinal Data Analysis, pp. 143-165. Boca Raton: Chapman & Hall/CRC Press.

Muthén, B. & Shedden, K. (1999). Finite mixture modeling with mixture outcomes using the EM algorithm. Biometrics, 55, 463-469. (#78)

Muthén, B., Brown, C.H., Masyn, K., Jo, B., Khoo, S.T., Yang, C.C., Wang, C.P., Kellam, S., Carlin, J. & Liao, J. (2002). General growth mixture modeling for randomized preventive interventions. Biostatistics, 3, 459-475. (#87)

Muthén, B. (2004). Latent variable analysis: Growth mixture modeling and related techniques for longitudinal data. In D. Kaplan (ed.), Handbook of quantitative methodology for the social sciences (pp. 345-368). Newbury Park, CA: Sage Publications. (#100)

Further Readings On General Growth Mixture Modeling (Continued)

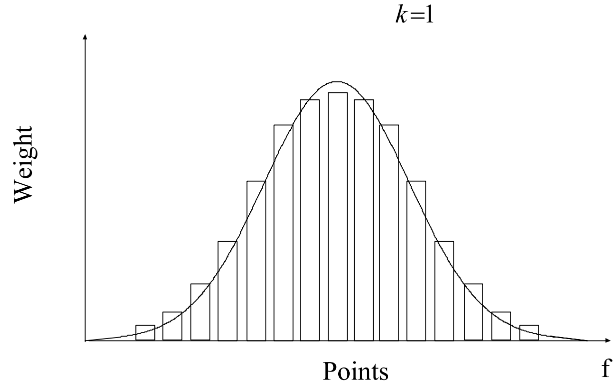
Muthén, B., Khoo, S.T., Francis, D. & Kim Boscardin, C. (2002). Analysis of reading skills development from Kindergarten through first grade: An application of growth mixture modeling to sequential processes. In S.R. Reise & N. Duan (eds), Multilevel modeling: Methodological advances, issues, and applications (pp. 71 – 89). Mahaw, NJ: Lawrence Erlbaum Associates. (#77)

Numerical Integration

Numerical Integration
With A Normal Latent Variable Distribution

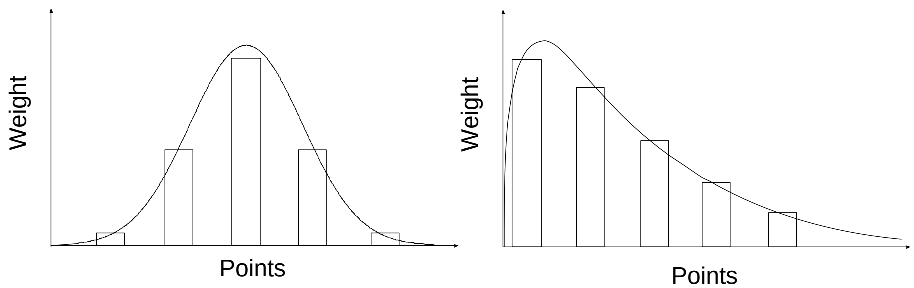
Numerical integration approximates the integral by a sum

$$[y] = \int [f][y \mid f]\, df = \sum_{k=1}^{K} w_k [y \mid f_k]$$



Fixed weights and points

Non-Parametric Estimation Of The Random Effect Distribution



Estimated weights and points
(class probabilities and class means)

Numerical Integration

Numerical integration is needed with maximum likelihood estimation when the posterior distribution for the latent variables does not have a closed form expression. This occurs for models with categorical outcomes that are influenced by continuous latent variables, for models with interactions involving continuous latent variables, and for certain models with random slopes such as multilevel mixture models.

When the posterior distribution does not have a closed form, it is necessary to integrate over the density of the latent variables multiplied by the conditional distribution of the outcomes given the latent variables. Numerical integration approximates this integration by using a weighted sum over a set of integration points (quadrature nodes) representing values of the latent variable.

Numerical Integration (Continued)

Numerical integration is computationally heavy and thereby time-consuming because the integration must be done at each iteration, both when computing the function value and when computing the derivative values. The computational burden increases as a function of the number of integration points, increases linearly as a function of the number of observations, and increases exponentially as a function of the dimensions of integration, that is, the number of latent variables for which numerical integration is needed.

Practical Aspects Of Numerical Integration

- Types of numerical integration available in Mplus with or without adaptive quadrature
  - Standard (rectangular, trapezoid) – default with 15 integration points per dimension
  - Gauss-Hermite
  - Monte Carlo
- Computational burden for latent variables that need numerical integration
  - One or two latent variables — Light
  - Three to five latent variables — Heavy
  - Over five latent variables — Very heavy

Practical Aspects Of Numerical Integration (Continued)

- Suggestions for using numerical integration
  - Start with a model with a small number of random effects and add more one at a time
  - Start with an analysis with TECH8 and MITERATIONS=1 to obtain information from the screen printing on the dimensions of integration and the time required for one iteration and with TECH1 to check model specifications
  - With more than 3 dimensions, reduce the number of integration points to 5 or 10 or use Monte Carlo integration with the default of 500 integration points
  - If the TECH8 output shows large negative values in the column labeled ABS CHANGE, increase the number of integration points to improve the precision of the numerical integration and resolve convergence problems

Technical Aspects Of Numerical Integration

Maximum likelihood estimation using the EM algorithm computes in each iteration the posterior distribution for normally distributed latent variables f,

$$[\, f \,|\, y \,] = [\, f \,]\, [\, y \,|\, f \,] \,/\, [\, y \,], \qquad (97)$$

where the marginal density for $[y]$ is expressed by integration

$$[\, y \,] = \int [\, f \,]\, [\, y \,|\, f \,]\, df. \qquad (98)$$

- Numerical integration is not needed with normally distributed, continuous y – the posterior distribution is normal

Technical Aspects Of Numerical Integration (Continued)

- Numerical integration is needed:
  - Categorical outcomes u influenced by continuous latent variables f, because $[u]$ has no closed form
  - Latent variable interactions f x x, f x y, f_1 x f_2, where $[y]$ has no closed form, for example

$$[\,y\,] = \int [\,f_1\,, f_2\,]\,[\,y | f_1,\, f_2,\, f_1 f_2\,]\, df_1\, df_2 \qquad (99)$$

  - Random slopes, e.g. with two-level mixture modeling

Numerical integration approximates the integral by a sum

$$[\,y\,] = \int [\,f\,]\,[\,y\,|\,f\,]\,df = \sum_{k=1}^{K} w_k\,[\,y\,|\,f_k\,] \qquad (100)$$

Further Readings On Random Effects, Numerical Integration, And Non-Parametric Representation Of Latent Variable Distributions

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